Functional Forms and Specification Errors

Eco321: Econometrics

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Functional Forms

Dummy variables

Specification Errors

Example



Functional Forms

Quadratic Functional Forms¹

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$$

Maginal effect of x on y

$$\frac{dy}{dx} = \beta_1 + \beta_2 x$$

► Concave (β₂ < 0) or canvex (β₂ > 0) function

$$\frac{d^2y}{dx^2} = \beta_2$$

¹the subscript *i* is suppressed for simplicity

Functional Forms

Log-linear Functional Form

$$y = A x^{\beta_1} e^{\epsilon}$$

Taking natural logarithm in both side of the equation

 $\log y = \beta_0 + \beta_1 \log x + \epsilon$

► $\beta_0 = \log A$

• The parameter β_1 is (constant) elasticity

$$\frac{d(\log y)}{d(\log x_1)} = \frac{dy}{dx_1} \frac{x_1}{y} = \beta_1$$

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Semilog Functional Form²

 $\log y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

• β_1 is proportionate change in y per unit change in x_1 .

$$\frac{\partial y}{\partial x_1} \frac{1}{y} = \frac{\partial (\log y)}{\partial x_1} = \beta_1$$

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It is widely used in human capital models

²See page 166 in textbook for more details

Dummy Variables

Example

$Wage = \beta_0 + \beta_1 Education + \beta_2 Female + \epsilon$

- ► The dummy variable takes only two values, 1 or 0.
- Female = 1 if the person is woman
 0 if otherwise (the person is man)
- The Female variable measures the effect of gender on wages
- The statistical significance of the estimator β₂ shows the existence of gender discrimination

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Dummy Variables

Analysis

- ► For Woman Wage = $(\beta_0 + \beta_2) + \beta_1$ Education + ϵ
- For Man Wage = $\beta_0 + \beta_1$ Education + ϵ
- ▶ β₂: wage difference
- ► Assume that β₂ < 0</p>



Dummy variables

Dummy Variable with Interaction term

Example

 $Wage = \beta_0 + \beta_1 Education + \beta_2 Female + \beta_3 (Female \cdot Education) + \epsilon$

- ► For Woman Wage = $(\beta_0 + \beta_2) + (\beta_1 + \beta_3)$ Education + ϵ
- ► For Man Wage = $\beta_0 + \beta_1$ Education + ϵ
- β₂: wage difference
- β_3 : different marginal effect
- Assume that $\beta_2 < 0$ and $\beta_3 > 0$



Education

Omitted Variable

True Model

 $y_i = \gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + u$

Estimated Model

 $y = \beta_0 + \beta_1 x_1 + \epsilon$

▶ The omission of relevant variable *x*₂ (omitted variable)

Result

The OLS estimator is biased unless x_1 and x_2 are not correlated (Show it!!)

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Omitted Variable

Meaning of β_1 : Mixture of Direct and Indirect Effects Assume that

 $x_2 = \delta_0 + \delta_1 x_1 + \nu$

Insert it in the true model and rearrange it

 $y = (\gamma_0 + \gamma_2 \delta_0) + (\gamma_1 + \gamma_2 \delta_1) x_1 + (\gamma_2 \nu + u)$

 $\blacktriangleright \ \beta_1 = \gamma_1 + \gamma_2 \delta_1$

- γ_1 : direct effect of x_1 on y
- $\gamma_2 \delta_1$: indirect effect of x_1 on y through x_2
- β_1 and γ_1 may have different sign³

Specification Errors

Measurement Error

Measurement Error in Explanatory Variables

Model

$$y = \beta_0 + \beta_1 x^* + \epsilon$$

► x^{*} in not observed.

▶ Instead *x* is observed but with error (errors-in-variables).

Measurement Error u

$$x = x^{*} + u$$

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Assume that x and u are correlated

Measurement Error

Result

The OLS estimator is biased toward zero (attenuation bias)

The correlation between regressor and error term

- It means violation of the $E(\epsilon|x) = 0$ assumption (Show it!!)
- The OLS estimator is biased

Remedy

Instrumental variable (IV) estimator⁴

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⁴It will be covered later

Proxy Variables

The Effect of Education on Wage

Wage = $\beta_0 + \beta_1$ Education + $X\beta_2 + \epsilon$

The Education variable is not measured

Proxy for Education: Years of Schooling

The error *u*

Years of Schooling = Education + u

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If years of schooling and the error term u is correlated, the OLS estimator is biased

Example: The effect of education of wage

The Econometric Model⁵

 $log(wage) = \beta_0 + \beta_1 Age + \beta_2 Education + \beta_2 Female + \beta_2 Africamer + \epsilon$

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The things to be considered

- Semilog Functional form
- The Female and Africamer dummy variables
- ▶ The possible omitted variables: experience, ability
- Proxy for education: years of schooling