Heteroscedasticity and Autocorrelation

Eco321: Econometrics

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Spring 2005

Spherical and Non-spherical Disturbances

Heteroscedasticity

Autocorrelation (Serial Correlation)

The Classical Regression Model

Model

$$y = X\beta + \epsilon \text{ with } E(\epsilon|X) = 0 \text{ and } E(\epsilon\epsilon^{'}|X) = \sigma^2 I$$

$$E(\epsilon\epsilon^{'}) = \begin{bmatrix} E(\epsilon_1^2) & E(\epsilon_1\epsilon_2^{'}) & \dots & E(\epsilon_1\epsilon_n^{'}) \\ E(\epsilon_2\epsilon_1^{'}) & E(\epsilon_2^2) & \dots & E(\epsilon_2\epsilon_n^{'}) \\ E(\epsilon_3\epsilon_1^{'}) & & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ E(\epsilon_n\epsilon_1^{'}) & E(\epsilon_n\epsilon_2^{'}) & \dots & E(\epsilon_n^2) \end{bmatrix} = \begin{bmatrix} \sigma^2 & 0 & \dots & 0 \\ 0 & \sigma^2 & \dots & 0 \\ 0 & & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^2 \end{bmatrix}$$

The Model with Spherical Disturbances

Homoscedasticity

- $Var(\epsilon_i|X) = E(\epsilon_i^2|X) = \sigma^2$ for all $i = 1, 2, ..., n$
- The error terms have the same variance for all observations

Non-autocorrelation

- $Cov(\epsilon_i, \epsilon_j|X) = E(\epsilon_i\epsilon_j^{'}|X) = 0$ for $i \neq j$
- An error term is not correlated with other error terms

The Model with Non-spherical Disturbance

Model

$$y = X\beta + \epsilon \text{ with } E(\epsilon|X) = 0 \text{ and } E(\epsilon\epsilon^{'}|X) = \Sigma = \sigma^2 V$$

Properties of OLS estimator

- The OLS estimator remains unbiased and consistent
- The OLS estimator is inefficient (not BLUE)[1]
- The OLS standard errors are incorrect and the test statistics based on them are incorrect

[1] Generalized Least Square (GLS) is BLUE

Heteroscedasticity

Model

$$y = X\beta + \epsilon \text{ with } E(\epsilon|X) = 0 \text{ and } E(\epsilon\epsilon^{'}|X) = \Sigma = \sigma^2 V$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ 0 & & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n^2 \end{bmatrix}$$

Heteroscedasticity

The heteroscedastic error terms

- The error terms have different variance: $E(\epsilon_i^2|X) = \sigma_i^2$
- The variance of the dependent variable y: $var(y|X) = \sigma_i^2$
- The variation of wage increases with education
  Assume that $\sigma_i^2 = \sigma^2 \cdot \mathrm{Education}_i$
- It is common in cross-section data

Test for Heteroscedasticity

The white test[2]

1. Regress y on X by OLS and obtain residuals e_i
2. An auxiliary regression of e_i^2 on all regressors, squares of regressors, (and cross products of regressors).
3. Compute nR^2 from step 2.
4. H_0: No heteroscedasticity

$$nR^2 \sim \chi_q^2$$

where q is the number of regressors in step2 less one

[2] When heteroscedasticity is of unknown form

Possible Remedies for Heteroscedasticity

Possible Remedies

- OLS with heteroscedasticity-consistent standard errors
- Generalized Least Squares(GLS) estimator[3]

[3] The GLS is called Weighted Least Squares(WLS) estimator when it is applied in the heteroscedasticity case.

Autocorrelation

The serially correlated error terms

- $E(\epsilon_i \epsilon_j^{'}|X) \neq 0$ for $i \neq j$
- An error term is correlated with other error term
- It is common in time-series data, especially macroeconomic data
- The disturbance is often called a shock

Autocorrelation

The 1st-order autoregressive process: AR(1) process

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t$$
$$\epsilon_t = \rho\epsilon_{t-1} + u_t$$
$$\text{where } u \sim iid(0, \sigma_u^2) \text{ and } |\rho| < 1$$

$$\Sigma = \sigma_\epsilon^2 \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \dots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & & \dots & 1 \end{bmatrix} \text{ where } \sigma_\epsilon^2 = \frac{\sigma_u^2}{1-\rho^2}$$

Autocorrelation

Durbin-Watson test: test for AR(1) disturbances

- The test statistic

$$\mathrm{DW} = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=2}^{n}(e_t)^2}$$

- if there is no AR(1), DW is close to 2.
- The Lower and Upper Critical values[4]
  If DW<LC, we reject the null hypothesis
  If DW>UC, we accept the null hypothesis
  If LC<DW<UC, the test is inconclusive

[4] The Appendix E shows the critical values

Autocorrelation

Possible Remedies

- Cochrane-Orcutt transformation (Partial/Quasi Differencing)[5]
- Generalized Least Squares (GLS) estimator

[5] See p. 228 in the textbook