Review for Midterm

Description of real and complex numbers as fields. Complex conjugate. Absolute value of a complex number.

Definition of a vector space over a field; e. g., the real or complex numbers.

Subspaces, sums and direct sums.

Span, linear independence, bases and dimension.

Prove that if a space has n linearly independent vectors, then a spanning set must have at least n elements. It follows that any two bases have the same number of elements.

Linear maps, their null spaces and ranges. The rank plus nullity theorem and its proof.

The matrix of a linear map. Matrix multiplication and composition of linear maps.

Invertible maps and their matrices. Injective and surjective maps.

Given a linear map $T : V \to W$, find bases of V and W that make the matrix of T as simple as possible. It should have r ones and the rest zeros, where r is the rank of the map.

Equivalence relations, equivalence classes and partitions of a set. Show that an equivalence relation on a set gives a partition of the set and vise versa.

Direct sums, products and quotients of vector spaces.

The quotient map $\pi : V \to V/U$.

Linear functionals, dual spaces and the dual of a linear map. Show that the $(TS)' = S'T'$ where $'$ means "dual".

Subspaces and their annihilators. Dimension of an annihilator. Rank and nullity of the dual of a linear map.

Bases and dual bases. Given bases of V and W, and dual bases of V' and W', and a linear map $T : V \to W$ what is the relation between the matrix of T and the matrix of T'?

Polynomials with real or complex coefficients, or with coeffieients in a finite field. The degree of a polynomial. The division algorithm for polynomials. Prove that r is a root of a polynomial iff $(x - r)$ divides the polynomial.

The distinction between polynomials and polynomial functions, and its importance if the coefficients are in a finite field.

The fundamental theorem of algebra for polynomials with complex coefficients. (You needn't know the proof.)