# Review for Final

Everything that is on the Midterm exam review sheet.

Definition of an inner product for real and complex vector spaces.

Schwartz inequality and its proof

Triangle inequality

Parallelogram equality

Norms and how an inner product gives rise to a norm

Orthogonal and orthonormal sets, orthonormal bases

Gram-Schmidt procedure to get an orthonormal set

If an operator from $V$ to itself has a basis for which it's matrix is upper trianglular, then it has an orthonormal basis for which its matrix is upper triangular.

Subspaces and their perpendicular subspaces.

Orthogonal projections and the closest point in a subspace

Linear functionals

Let $V$ be a finite dimensional vector space. Then for any linear functional $\phi : V \to \mathbf{F}$, there exists a vector $w \in V$ such that $\phi(v) =< v, w >$ for all $v \in V$. Know how to prove this and show by example that it may not be true if $V$ is not finite dimensional.

Definition of the adjoint $T^*$ of a linear transformation $T$.

The matrix of $T^*$ with respect to orthonormal bases is the conjugate of the transpose of the matrix of $T$

Self-adjoint and normal operators

Self-adjoint operators have only real eigenvalues and they have an orthonormal basis of eigenvectors.

$T$ is normal iff $Tv$ and $T^*v$ have the same norm for all vectors v.

If $T$ is normal and $\lambda$ is an eigenvalue of $T$, then $\bar{\lambda}$ is an eigenvalue of $T^*$ with the same eigenvector.

Spectral theorem for normal operators on a complex vector space and for self-adjoint operators on a real vector space

Invariant subspaces of an operator and block diagonal matrices

Positive operators and their square roots

Isometries

Polar and Singular value decompositions

Traces and their properties