MAT123

Pythagorean Theorem

Pythagorean Identity

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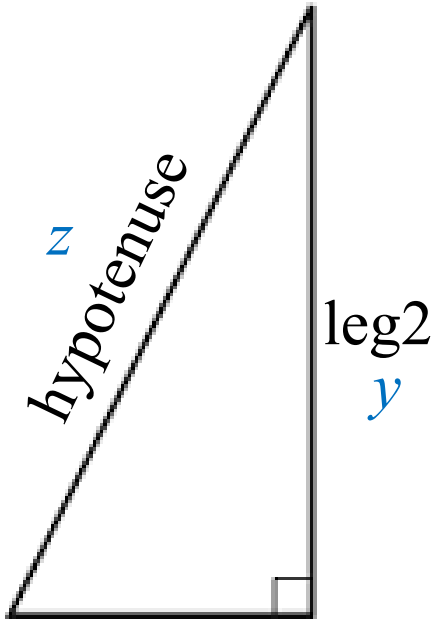
Pythagorean Theorem

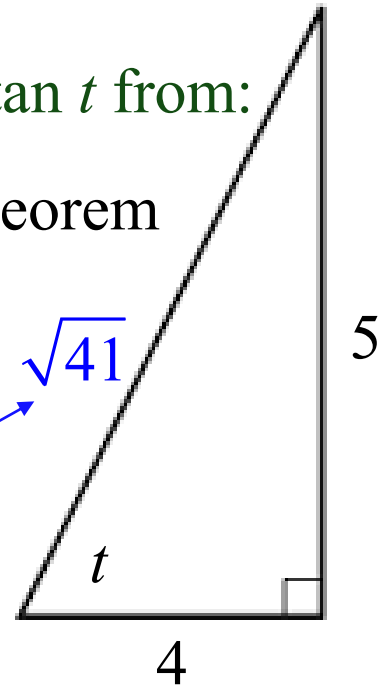
Used to solve for the 3rd side of a **RIGHT** triangle given other 2 sides.



$$(leg1)^2 + (leg2)^2 = (hypotenuse)^2$$

$$x^2 + y^2 = z^2$$

ex. Find sin *t*, sec *t* and tan *t* from:



Step 1: find hypotenuse using Pythagorean Theorem

$$5^2 + 4^2 = z^2$$
$$25 + 16 = z^2$$
$$41 = z^2$$
$$z = \pm\sqrt{41}$$

(take only + root)

Step 2: use trig ratios

$$\sin t = \frac{O}{H} = \frac{5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}}$$

rationalize

$$= \boxed{\frac{5\sqrt{41}}{41}}$$

$$\sec t = \frac{H}{A}$$

reciprocal of cos *t*

$$= \boxed{\frac{\sqrt{41}}{4}}$$

$$\tan t = \frac{O}{A}$$

$$= \boxed{\frac{5}{4}}$$

Pythagorean Identity

Useful for solving trig expressions and equations.

Based on Pythagorean Theorem:

$$x^2 + y^2 = z^2$$

$\cos t = x$, $\cos^2 t = x^2$

$\sin t = y$, $y^2 = (\sin t)^2$

can also be written: $\sin^2 t$

$y^2 + x^2 = 1^2$ * based on unit circle

$$\sin^2 t + \cos^2 t = 1$$

ex. Given $\sin\theta = \frac{4}{9}$, determine $\cos\theta$.

Use $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{4}{9}\right)^2 + \cos^2\theta = 1$$

$$\frac{16}{81} + \cos^2\theta = 1$$

solve for $\cos^2\theta$

$$\cos^2\theta = 1 - \frac{16}{81}$$ LCD is 81

$$\cos^2\theta = \frac{81}{81} - \frac{16}{81}$$

$$\cos^2\theta = \frac{65}{81}$$

$$\cos\theta = \sqrt{\frac{65}{81}} = \frac{\sqrt{65}}{\sqrt{81}} = \boxed{\frac{\sqrt{65}}{9}}$$