Math 319/320 Worksheet 2

Problem 1. Fill in the blanks in the following proof that
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

If $x \in A \cup (B \cap C)$ then either $x \in A$ or $x \in B \cap C$. If $x \in A$ then $x \in A \cup B$ and $x \in$ __________ and so $x \in$ _______________. On the other hand, if $x \in B \cap C$ then $x \in$ __________ and $x \in$ __________. Hence $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Now suppose that $x \in$ __________________________. Then $x \in$ __________ and $x \in$ __________.

If $x \in A$ then

On the other hand if $x \notin A$ then

Therefore

Problem 2. It is possible to take intersections and unions of many sets $A_i, i \in I$, not just two. We define

$$\cup_{i\in I} A_i := \{x : \exists i \in I \text{ such that } x \in A_i\}, \qquad \cap_{i\in I} A_i := \{x : x \in A_i \forall i \in I\}.$$

The set I is called the *indexing set*. Often it is the set of the first n integers $\{1, \dots, n\}$, but sometimes it is the infinite set $\mathbb{N}$ of all positive integers.

(i) Find three subsets A_1, A_2, A_3 of the plane $\mathbb{R}^2$ such that each double intersection $A_i \cap A_j$ is nonempty but the triple intersection $A_1 \cap A_2 \cap A_3$ is empty.

(ii) Find open intervals $A_i = (a_i, b_i) \subset \mathbb{R}$ such that each finite intersection $\cap_{1\le i\le n} A_i$ is nonempty but the infinite intersection $\cap_{i\in\mathbb{N}} A_i$ is empty.

Problem 3. Let $f : A \to B$ be a function and $C \subset A, D \subset B$. Show that $C \subset f^{-1}(f(C))$ and $f(f^{-1}D) \subset D$.

If f is injective, do either of these inclusions become equalities?

What if f is surjective?

Problem 4. Let A, B be subsets of a universal set U. Simplify the following expressions. You can draw Venn diagrams to help you.

(i) $(A \cap B) \cup (U \setminus A)$

(ii) $A \cup [B \cap (U \setminus A)]$