Math 319/320 Worksheet 1

Name: **ID:**

The first part of this worksheet is a review of basic logic. Recall the symbols

$$\forall = \text{ for all}, \quad \exists = \text{ there exists}, \quad \wedge = \text{ and}, \quad \vee = \text{ or}, \quad \sim = \text{ not}.$$

Remember also that if P and Q are statements the implication $P \Rightarrow Q$ ("P implies Q") is true if P is false and also if P is true and Q is true. Therefore its negation $\sim(P \Rightarrow Q)$ is logically equivalent to $P \wedge (\sim Q)$, i.e. $\sim(P \Rightarrow Q)$ holds iff P is true and Q is false.

Note: this is a slightly edited verion of the sheet that was handed out.

Problem 1. Negate the following statement: "If your glass is half-empty, you are a pessimist or you are thirsty." (Answer in words.)

Problem 2. The context in this problem is the set of all human beings. Let $E(x)$ be "x is educated," $F(x)$ be "x is female" and $O(x)$ be "x is older than 30." Then the statement "every uneducated male is older than 30" can be expressed as

$$\forall x, \Big((\sim E(x) \wedge \sim F(x)) \Longrightarrow O(x) \Big)$$

Express the following statements in a similar way:

(i) Some educated people are younger than 30.

(ii) Every female who is older than 30 is educated.

(iii) No uneducated person is both female and older than 30.

Problem 3. Consider the statement

"For every natural number n, if n^2 is even, then n is even."

Prove this statement in two different ways: (i) by showing that its contrapositive is true; (ii) by showing that its negation is false.
Note: the contrapositive of $P \Rightarrow Q$ is $\sim Q \Rightarrow \sim P$.

Problem 4. On a bumper sticker, I saw the statement

"For every real number x, there is a real number t such that $t(1-t) > x$."

After some thought I conjectured that this statement must be __________. To prove my conjecture carefully, I found a real number ____ such that for every real number ____ the inequality ____________________ held.

Hint: To understand the behavior of the function $f(t) = t(1-t)$ it might be helpful to draw its graph.