

Math 319 Midterm I

September 29, 2005

Name:

School ID:

Answer all the following questions, justifying all your statements. Each question is worth 10 points. There are five questions. Good luck!

Problem 1. (i) Show that if $f : A \rightarrow B$ is injective and $E \subset A$ then $f^{-1}(f(E)) = E$.

(ii) Give an example to show that equality need not hold if f is not injective.

1	
2	
3	
4	
5	
Total	

Problem 2. Prove the following statement by induction:

$$3 + 11 + \cdots + (8n - 5) = 4n^2 - n, \quad \text{for all } n \in \mathbb{N}.$$

(or we might ask

$$1^2 + 3^2 + \cdots + (2k - 1)^2 \leq 2k^3, \quad k \geq 1$$

I think an inequality is harder than an equality; so perhaps we should do the first since there isn't much time on this exam.)

Problem 3. (i) State the Archimedean property of the real numbers.

Use it to prove that if $x > 0$ is any real number then there is a rational number r such that $x > r > 0$.

Problem 4. (i) Let S be a subset of \mathbb{R} . Give the definitions of a lower bound of S and of $\inf S$.

(ii) If possible, give examples of sets S with the following properties. If there is no such example, give a brief explanation of why.

(a) a set $S \subset \mathbb{R}$ with no lower bound.

(b) a set $S \subset \mathbb{R}$ with a lower bound but no infimum.

Problem 5. ??