Math 319 Midterm I
September 29, 2005

Problem 1. (i) Show that if $f : A \to B$ is injective and $E \subset A$ then $f^{-1}(f(E)) = E$.
(ii) Give an example to show that equality need not hold if f is not injective.

Problem 2. Prove the following statement by induction:

$$3 + 11 + \cdots + (8n - 5) = 4n^2 - n, \quad \text{for all } n \in \mathbb{N}.$$

Problem 3. (i) State the Archimedean property of the real numbers.
Use it to prove that if $x > 0$ is any real number then there is a rational number r such that $x > r > 0$.

Problem 4. (i) Let S be a subset of $\mathbb{R}$. Give the definitions of a lower bound of S and of $\inf S$.

(ii) If possible, give examples of sets S with the following properties. If there is no such example, give a brief explanation of why.

(a) a set $S \subset \mathbb{R}$ with no lower bound.

(b) a set $S \subset \mathbb{R}$ with a lower bound but no infimum.

Problem 5. Let us say that a set S has property F if every injection $f : S \to S$ is surjective.
(i) Show that $\mathbb{N}$ does not have property F.
(ii) Show that if S has property F and $T \subset S$ then T has property F.