# Math 319 Homework 8

## Due Thursday, November 3, 2005

As always, justify all your answers.

**Problem 1.** (a) Give an example of a sequence that is bounded above but not bounded below and that has a convergent subsequence.

(b) Explain how to construct a monotonic increasing sequence of rational numbers that converges to $\sqrt{3}$.

**Problem 2.** Consider the sequences

$$x_n = \sin\left(\frac{n\pi}{4}\right), \quad y_n = \frac{1}{\sqrt{n}}, \quad z_n = x_n y_n.$$

(a) Write down the first 10 terms of $(x_n)$. Find two monotonic subsequences of $(x_n)$ with different limits.

(b) Which terms in the product sequence $(z_n)$ are peaks? (It might help to work out the first 10 terms of $z_n$.) Find a subsequence of $(z_n)$ that is strictly decreasing (i.e. $z_{n_{k+1}} < z_{n_k}$ for all $k$), and another that is strictly increasing.

(c) Does $(z_n)$ converge? (Explain which theorems you use in your argument.)

**Problem 3.** (a) Suppose that $x_n \geq 0$ for all $n$ and that $\lim x_n = 2$. Find a subsequence of $((-1)^n x_n)$ that converges to 2 and another that converges to $-2$. Does $((-1)^n x_n)$ converge?

(b) Suppose that $x_n \geq 0$ for all $n$ and you are told that $((-1)^n x_n)$ converges. Show that $(x_n)$ converges. What is its limit?

**Problem 4.** Describe all cluster points of $A$ where
a) $A = \mathbb{Z} \cup (0, 1)$,
b) $A = \{1/n : n \in \mathbb{N}\}$.

**Problem 5.** (a) Consider the intervals $I_k = (1 + \frac{1}{3k+1}, 1 + \frac{1}{3k})$ for $k = 1, 2, 3, 4, \ldots$. Write down $I_1, I_2$ and $I_3$ explicitly.

(b) Make an accurate sketch of the set $A = \cup_{k \geq 1} I_k$.

(c) Describe all cluster points of $A$.