Math 319 Homework 7

Due Thursday, October 27, 2005

Problem 1. Suppose that (x_n) is a sequence with limit 3. Prove carefully from the definition that

$$\lim \frac{1}{x_n} = \frac{1}{3}.$$

Note: I do NOT want you to quote Theorem 3.2.3(b). Instead, adapt the proof to this case.

Problem 2. Suppose that the sequences (x_n) and (z_n) both converge to w and that $x_n \leq y_n \leq z_n$ for all n.

(i) Use Theorem 2.1.10 to show that the sequence $(y_n - x_n)$ converges to 0.

(ii) Use (i) and the sum theorem (3.2.3(a)) to conclude that (y_n) converges to w.
Note This is known as the **Squeeze Theorem.** The book gives a different (more direct) proof in 3.2.7.

Problem 3. Find the limits, justifying your answer carefully. (Use any theorems you like, but say what you are using.)

(i) $x_n = \dfrac{4n^2 - n + 1}{n^2 - 3n}$.

(ii) $(-1)^n \dfrac{n+1}{n^2+2}$.

Problem 4. (i) Show that the sequence $x_n = (n^2 - n)/(n+1)$ is monotonic.

(ii) Define x_n inductively by the relation $x_{n+1} = \frac{1}{2}(x_n + 5/x_n)$. Assume that $x_n \geq x_{n+1} > 0$ for all n. Then (x_n) converges by Theorem 3.3.2. What is its limit?

Problem 5. Consider the sequence defined inductively by: $x_{n+1} = 4 - 3/x_n$.

(i) Show that if $1 < x_1 < 3$ then this is monotonic increasing. What is the limit?

(ii) Show that if $x_1 > 3$ then this is monotonic decreasing. What is the limit?

(iii) Is there a value of x_1 that gives a sequence with limit 1?

Note The points 1 and 3 are special because they are the roots of the quadratic equation $x^2 - 4x + 3 = 0$.