

Math 319 Homework 6

Due Thursday, October 20, 2005

Problem 1. (i) Let $x_n = \frac{1}{4n-3}, n \geq 1$. Find an integer K such that

$$x_n < \frac{1}{35}$$

for all $n \geq K$. Explain your reasoning.

(ii) Same as (i) with $x_n = \frac{1}{4^n}$.

Hint: use the calculations in 3.1.11 (b).

Problem 2. Use the definition of limit to prove that:

(i) $\lim \frac{n}{2n-1} = \frac{1}{2}$.

(ii) $\lim \frac{\sqrt{n}}{n+1} = 0$.

Problem 3. Prove Bernoulli's inequality:

if $x + 1 > 0$ then $(x + 1)^n \geq 1 + nx$ for all $n \geq 1$.

Hint: Prove this by induction on n . Make clear in your argument why you need to assume $x + 1 > 0$.

Problem 4. Show that the sequence $n^2 - \sin n$ has no limit.

Problem 5. (i) Suppose that $\lim z_n = z$ where $z \neq 0$. Show that there is $K \in \mathbb{N}$ such that $z_n \neq 0$ for all $n \geq K$.

(ii) Suppose that $\lim z_n = z$ where $z \neq 0$ and $z_n \neq 0$ for all n . Show there is $\delta > 0$ such that $|z_n| > \delta$ for all n .