Math 319/320 Homework 3

Due Thursday, September 22, 2005
revised version

Problem 1. Show that
$$\left[\frac{1}{2}(a+b)\right]^2 \le \frac{1}{2}(a^2+b^2)$$
for all $a, b \in \mathbb{R}$. Show that equality holds if and only if $a = b$.

Problem 2. Assume that $a < x < b$ and $a < y < b$. Show that $|x - y| \le b - a$. Find a geometric explanation for the obtained inequality.

Problem 3. Let $a, b \in \mathbb{R}$ and $a \neq b$. Show that there exist ϵ-neighborhood $U_\epsilon(a)$ of a and ϵ-neighborhood $V_\epsilon(b)$ of b such that $U_\epsilon(a) \cap V_\epsilon(b) \neq \emptyset$.

Problem 4. Let $S := \{x \in \mathbb{R} | x \ge 0\}$. Show that S has lower bounds, but no upper bounds. Show that $\inf\ S = 0$.

Problem 5. If $S \subset \mathbb{R}$ contains one of its upper bounds, then this upper bound is the supremum of S.