Math 203 - Fall 2018
Practice Problems for the Second Exam

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1. A function $z = f(x, y)$ is defined implicitly by the equation
$$z^2(1 + x^2y^2) - 8ze^{y^2-4x} = y - 2x + 4,$$
and it is known that $f(1, 2)$ is positive. Compute $\frac{\partial z}{\partial x}$ at $(x, y) = (1, 2)$.

2. Consider the surface
$$S = \{(x, y, z) \ ; \ e^{xz} + (x^2 + y^2)z = 3\}.$$
   (a) Find the equation for the plane tangent to S at the point $(0, 1, 2)$.
   (b) Find the vector equation for the line perpendicular to S at the point $(0, 1, 2)$.
   (c) Let $z = f(x, y)$ be a function defined implicitly by the requirement that $(x, y, z) \in S$. Compute
$$\frac{\partial f}{\partial x}(0, 1) \quad \text{and} \quad \frac{\partial f}{\partial y}(0, 1).$$

3. Consider the function
$$f(x, y) = e^{2x^2(1+y)}.$$
   (a) Find the directional derivative of f at the point $(1, -1)$ along the direction $\mathbf{u}_o = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$.
   (b) Find the maximum value of $D_{\mathbf{u}} f(1, -1)$ among all unit vectors $\mathbf{u}$.

4. Find all of the critical points of the function
$$f(x, y) = \tfrac{6}{7}x^7 + 4(y^2 - 1)x - \tfrac{1}{2}x^4.$$

5. Find the absolute minimum and maximum values of the function
$$f(x, y) = (x - 1)^2 + y^2$$
on the filled-in ellipse
$$\mathbf{E} = \{(x, y) \ ; \ 2x^2 + y^2 \le 6\},$$
and the points at which the minimum and maximum are achieved.

6. Let (x, y, z) be coordinates in three dimensional space, with the vector $\langle 0, 0, 1\rangle$ defining the positive vertical direction (as usual). Find the highest point on the surface

$$(x-y)^2 + 2(y-z)^2 + (x+z)^2 = 10.$$

7. Compute the iterated integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} e^{x^2+y^2}\, dx dy.$$

8. A conical cup has radius 1.5 inches and height 4 inches. The conical cup is used to transfer water from a reservoir to a cylindrical cup of radius 3 inches and height 6 inches. How many trips are needed to completely fill the cylindrical cup?

9. Find the integral of the function

$$f(x, y, z) = \frac{x^2 z}{x^2 + y^2 + z^2}$$

over the half-ball

$$C: \quad x^2 + y^2 + z^2 \leq 4, \quad \text{and} \quad z \geq 0.$$

10. Find the mass of a conical ellipsoid

$$K = \{(x, y, z)\ ;\ z^2 \leq a^2x^2 + b^2y^2,\ 0 \leq z \leq 4\}$$

whose mass density is

$$\rho(x, y, z) = \frac{2z}{\sqrt{a^2x^2 + b^2y^2}}.$$