Homework 3. Solutions.

12.1 Problems 8, 16, 18, 19-22, 34

8. Domain is all real numbers $t > 0$.
16. Parametric Equations are

$$x(t) = -4t - 3$$
$$y(t) = 4t - 2$$
$$z(t) = -3t + 5$$

Vector valued function is $r(t) = (-4t - 3)\hat{i} + (4t - 2)\hat{j} + (-3t + 5)\hat{k}$

18. $r(t) \cdot u(t) = t^2(t - 2)$, a scalar-valued function.
19. B
20. C
21. A
22. D

34. See sketch at the end of file.

12.2 Problems 8, 18, 22, 44, 70

8. $r'(t) = \langle -e^{-t}, e^t \rangle$, $r'(t_0) = \langle -1, 1 \rangle$, $r(t_0) = \langle 1, 1 \rangle$. To obtain the graph note that $x = x(t) = e^{-t} = \frac{1}{y}$, since $y = y(t) = e^t$. See sketch at the end of file.

18. $r'(t) = \langle \dfrac{1}{\sqrt{1-t^2}}, \dfrac{-1}{\sqrt{1-t^2}}, 0 \rangle$

22. a) $r'(t) = \langle -8\sin t, 3\cos t \rangle$, b) $r''(t) = \langle -8\cos t, -3\sin t \rangle$, c) $r' \cdot r'' = 55 \sin t \cos t$

44. $\int r(t)dt = \langle \tan t + C_1, \arctan t + C_2 \rangle$

70. a) The curve is an ellipse. b) The maxima for $||r'||$ occurs at $0, \pi$, with maximum value at those points equal to 3 and the minima occur at the points $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$ with value at those points equal to 2. The max/min for $||r''||$ occur at the same points as for $||r'||$, but this time the value at the max. is 2 and value at the min is 3.

12.3 Problems 10, 20, 36, 38, 46

10. a) The velocity v, speed $||v||$ and acceleration a are given by $v = r'(t) = \langle -e^{-t}, e^t \rangle$
$||v|| = \sqrt{e^{-2t} + e^{2t}}$, $a = \langle e^{-t}, e^t \rangle$
b) $v(0) = \langle -1, 1 \rangle$, $a(0) = \langle 1, 1 \rangle$
c) See end of file for sketch.

20. a) $v(t) = r'(t) = \langle \dfrac{1}{t}, \dfrac{-2}{t^3}, 4t^3 \rangle$, $a(t) = r''(t) = \langle \dfrac{-1}{t^2}, \dfrac{6}{t^4}, 12t^2 \rangle$, $||v|| = \sqrt{\dfrac{1}{t^2} + \dfrac{4}{t^6} + 16t^6}$
b) $v(\sqrt{3}) = \langle \dfrac{-1}{\sqrt{3}}, \dfrac{-2}{(\sqrt{3})^3}, 4(\sqrt{3})^3 \rangle$, $a(\sqrt{3}) = \langle \dfrac{-1}{3}, \dfrac{2}{3}, 36 \rangle$

36. The position function for the bomb is given by

$$r(t) = (792\cos\theta)t\hat{\imath} + [30,000 + (792\sin\theta)t - \frac{1}{2}gt^2]\hat{\jmath}$$

where θ is the angle of elevation. To determine the time when the bomb should be released we first solve for t when the vertical component of $r(t)$ is identically zero, i.e. we need to solve the quadratic equation

$$30000 + (792\sin\theta)t - \frac{1}{2}gt^2 = 0$$

Solutions are given by

$$t = \frac{-792\sin\theta \pm \sqrt{792^2\sin\theta + 2g\cdot 30,000}}{-g}$$

Observe that of the two solutions, only $t_0 = \dfrac{-792\sin\theta - \sqrt{792^2\sin\theta + 2g\cdot 30,000}}{-g}$ is positive. Now, assuming that when the bomb is released $\theta = 0$, we obtain that $t_0 = \dfrac{\sqrt{60000g}}{g} = 61.24$, (with $g = 16ft/sec^2$). We can then determine how far the target is in the horizontal direction, since

$$x(t_0) = (792\cos\theta)t_0 = \frac{792\sqrt{60000g}}{g} = 48502$$

Using this we see that the angle of depression θ_1 must be given by

$$\tan(\theta_1) = \frac{30000g}{792\sqrt{60000g}} = \frac{30000}{48502} = 0.61$$

Therefore the bomb should be released at the time when the angle of depression θ_1 satisfies $\theta_1 = \arctan\left(\dfrac{30000g}{792\sqrt{60000g}}\right) = \arctan(0.61) = 32$ degrees. (See end of file for a sketch)

To determine the speed at the time of impact, note that

$$v(t) = \langle 792\cos\theta, 792\sin\theta - gt\rangle$$

so $||v|| = \sqrt{792^2 + g^2t^2}$. Since the bomb hits the target at time $t_0 = \dfrac{\sqrt{60000g}}{g}$, we get that the speed at the time of impact is

$$||v(t_0)|| = \sqrt{792^2 + 60000g} = 1259.9ft/sec$$

38. The horizontal component of the position vector is given by $x(t) = (v_0\cos\theta)t$. We are given that $\theta = 12$ degrees and we need to determine v_0 assuming that when $y(t) = 0, x(t) = 200$. Thus, we first determine the time at which $y(t) = 0$, this means $v_0\sin 12t - \frac{1}{2}gt^2 = t(v_0\sin(12) - \frac{1}{2}gt)$, so $t = \dfrac{2v_0\sin 12}{g}$, and then solve for v_0 in the equation

$$v_0\cos(12)\frac{2v_0\sin 12}{g} = 200$$

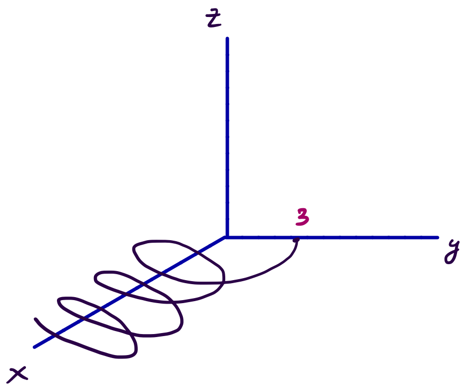
which gives

$$v_0 = \sqrt{\frac{200g}{\sin(24)}}$$

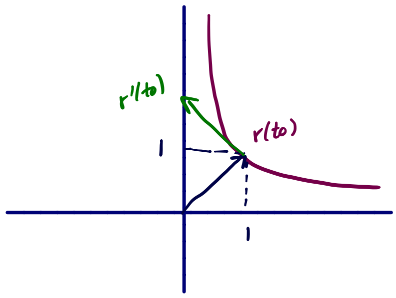
46. If $r(t) = b[\omega t - \sin(\omega t)]\hat{i} + b[1 - \cos(\omega t)]\hat{j}$, then $r'(t) = b\langle \omega - \omega\cos(\omega t), \omega\sin(\omega t)\rangle$, and $||v|| = ||r'|| = (b\sqrt{2}\omega)\sqrt{1 - \cos(\omega t)}$. The maximum for $||v||$ then occurs when $t = \frac{\pi}{\omega}$ (at which point $||v|| = 2b\omega$). Assuming $b = 1, \omega = 60$, we see that the speed of a point on the circumference of a tire is twice as much as that of the car.

SKETCHES

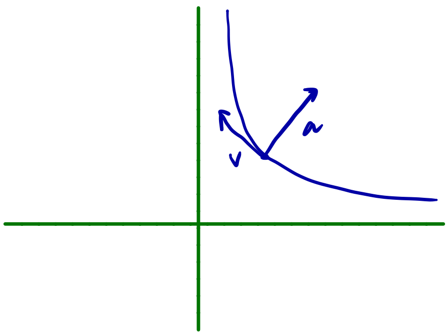
(34.)



(8.)



(10.) c)



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