Homework 1. Solutions.

11.1: Problems 8, 14, 36, 44, 82

8. The vector $\vec{u} = \langle 15, -3\rangle$ and $\vec{v} = \langle 15, -3\rangle$. The two vectors are equal since their first and second components coincide.

14. a). See end of file
b). Component form $\vec{v} = \langle -10, 0\rangle$
c). Linear combination $\vec{v} = -10\hat{i} + 0\hat{j}$
d.) See end of file.

36. $||\vec{v}|| = 5\sqrt{10}$ so $\vec{u} = \langle \frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\rangle$. Norm of $\vec{u} = \dfrac{\sqrt{1+9}}{\sqrt{10}} = 1$

44. See end of file for the graphs. $||u+v|| = ||\langle -2, 0\rangle|| = 2 \leq \sqrt{13} + \sqrt{5} = ||\vec{u}|| + ||\vec{v}||$

82 Denote by F_1, F_2 the tension on each rope. Since the resultant force is vertical,

$$100 = ||F|| = ||F_1|| \cos 30 + ||F_2|| \cos(20)$$

Moreover, the horizontal components must satisfy

$$||F_1|| \sin(20) = ||F_2|| \sin(30)$$

Solving the linear system for $||F_1||, ||F_2||$, we get

$$||F_1|| = 44.64lb, ||F_2|| = 65.27lb$$

The vector components of each force are thus

$$F_1 = \langle 65.27 \sin(20), 65.27 \cos(20)\rangle$$
$$F_2 = \langle 44.64 \sin(30), 44.64 \cos(30)\rangle$$

11.2: Problems 28, 32, 34, 46, 98

28 $d = \sqrt{4+1+25} = \sqrt{30}$

32 If $\vec{u}$ is the vector with initial point $A = (4, -1, -1)$ and terminal point $B = (2, 0, -4)$ and $\vec{v}$ is the vector with initial point $A = (4, -1, -1)$ and terminal point $C = (3, 5, -1)$, then the lengths of the sides are $|AB| = ||\vec{u}|| = \sqrt{14}$, $|BC| = ||\vec{v}|| = \sqrt{35}$, $|AC| = ||\vec{v} + \vec{u}|| = \sqrt{37}$. The triangle is not isosceles because no two sides have the same length. Now, if the triangle was right, then Pythagoras theorem would imply that $|AB|^2 + |BC|^2 = |AC|^2$, (observe that necessarily the right angle would be opposite the side AC, which is the longest), but this is impossible since $35 + 14 \neq 37$

34 Midpoint is given by $\left(\frac{7-5}{2}, \frac{2-2}{2}, \frac{2-3}{2}\right) = \left(1, 0, -\frac{1}{2}\right)$

46

$$4\left(x^2-6x+9\right)-4\cdot 9+4(y^2-y+\frac{1}{4})-1+4(z^2+2z+1)-4-23=0$$
$$\left(x^2-6x+9\right)+(y^2-y+\frac{1}{4})+(z^2+2z+1)=\frac{64}{4}$$
$$(x-3)^2+(y-\frac{1}{2})^2+(z-1)^2=8^2$$

Center of the sphere is $(3,\frac{1}{2},-1)$ and radius is 8.

98 Component form is $F=\langle 75,50,-100\rangle$

11.3: Problems 8, 14, 24, 30, 58

8 a) $u\cdot v=10, u\cdot u=50, ||v||^2=6, (u\cdot v)v=\langle -10,20,10\rangle, u\cdot(3v)=3(u\cdot v)=30$

14 The angle is $\frac{3\pi}{4}-\frac{\pi}{6}=\frac{7\pi}{12}$ or 105 degrees. One can see this without using the formula (see Sketch at the end of the file), or by using sum-to-product trig. identities: since each vector has unit length,

$$\cos(\theta)=u\cdot v=\cos(\frac{\pi}{6})\cos(\frac{3\pi}{4})+\sin(\frac{\pi}{6})\sin(\frac{3\pi}{4})=\cos(\frac{3\pi}{4}-\frac{\pi}{6})$$

24 They're orthogonal since $u\cdot v=-2\cdot 2+3\cdot 1+(-1)\cdot(-1)=0$

30 As in 11.2 (32), $|AB|=||u||=||(-3,12,5)||=\sqrt{178}$, $|BC|=||v||=||(5,1,-9)||=\sqrt{107}$, $||AC||=||v+u||=\sqrt{189}$. Since $u\cdot v=-48<0$, triangle is obtuse.

58 The force exerted due to gravity is $F=-5400\hat{j}$. The force required to keep the vehicle from rolling down is

$$F_1=proj_vF$$

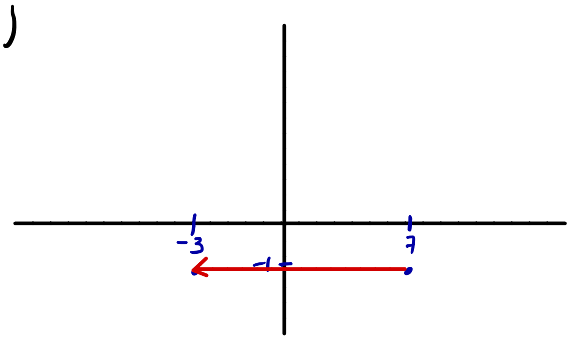
where $v=\cos(18)\hat{i}+\sin(18)\hat{j}$, which has unit length. Hence

$$F_1=(F\cdot v)v=(-5400\cdot\sin(18))(\cos(18)\hat{i}+\sin(18)\hat{j})$$

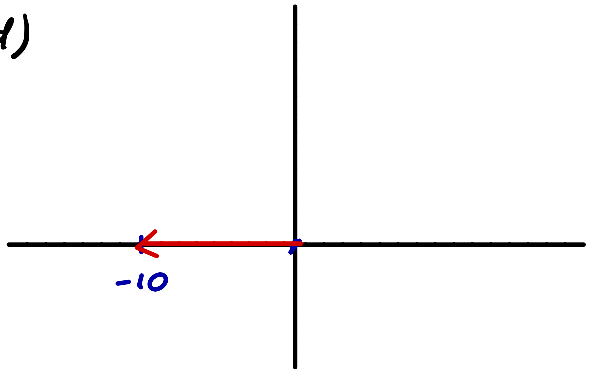
The force perpendicular to the hill is

$$F_2=F-F_1=5400\sin(18)\cos(18)\hat{i}+(-5400-5400\sin^2(18))\hat{j}$$

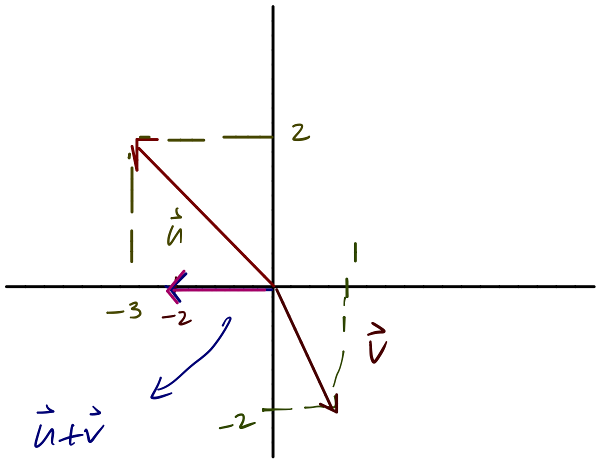
14 a)



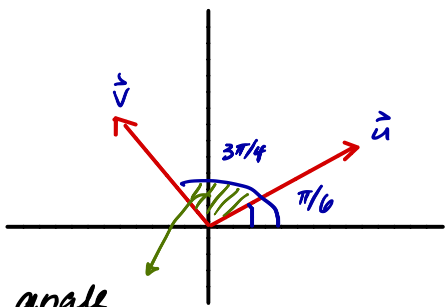
14 d)



44)



14)



angle
between $\vec{u}, \vec{v}$ is the difference.