Math 125 - Fall 2006
Practice Final Examination

1. Let f be a continuous function. Find
$$\lim_{x\to\infty} f\left((1-\tfrac{1}{x})^x\right).$$

2. Consider the equation $x + e^x = 0$. Is there a solution to this equation? Why or why not.

3. Find the derivative of the function
$$e^{2\tan(\sqrt{x})}.$$

4. Consider the function
$$f(x) = \begin{cases} \frac{\sin x}{x} & x < 0 \\ x^3 + 2x + 1 & x \geq 0 \end{cases}$$
At which points is f continuous? At which points is it differentiable?

5. Let $f(x) = x \ln\left(1 + e^{x^2}\right)$. Find $f'(5)$.

6. Show that the curves
$$e^{x^2-y^2}\cos(2xy) = 1 \quad \text{and} \quad e^{x^2-y^2}\sin(2xy) = 0$$
meet orthogonally at the point $(\sqrt{\pi}, \sqrt{\pi})$.

7. Find the derivative of the function
$$f(x) = \frac{(\sin x)^2(\tan x)^2}{(x^2+1)^2}.$$

8. Find an equation for the tangent line to the curve
$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2 = 0$$
through the point $(0, 0.5)$.

9. If $f(x) = e^x/(x+1)^3$, find $f'(x)$ and $f''(x)$.

10. Find the limit
$$\lim_{x\to 1} \frac{x^\pi - 1}{x^e - 1}.$$

11. Show that $e^x \geq 1 + x$ for $x \geq 0$. (Hint: Consider the function $f(x) = e^x - 1 - x$.)

12. A particle is moving along the curve $y = x^2$. As it passes through the point $(2, 4)$, its y coordinate changes at a rate of 5 m/sec. What is the rate of change of the particle's distance to the origin at this instant?

13. Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^2 - \ln x^2$$

on the interval $[1/4, 4]$.

14. Find

$$\lim_{x \to \frac{\pi}{2}} \tan(7x)\cos(4x).$$

15. A woman wants to get from a point A on the shore of a circular lake to a point C diametrically opposite A in the shortest possible time. She can walk at a speed of 4 mi/hr and row at a speed of 2 mi/hr. How should she proceed?

16. Consider the function

$$f(x) = x^3 - 7x^2 + 9x - \pi.$$

(i) Find all the critical points of f, and the values of f at those points. State weather these points are local maxima, local minima or neither.

(ii) Find all the inflection of points of f.