MAT 533 S25
PROBLEM SET 7

1. (Folland, Exercise 6.27) Let $p \in (1, \infty)$. Show that the operator

$$Tf(x) := \int_{\mathbb{R}} \frac{f(y)dy}{x+y}$$

satisfies $||Tf||_p \leq C_p ||f||_p$ where

$$C_p = \int_{\mathbb{R}} \frac{dx}{(1+x)x^{1/p}}.$$

2. (Folland, Exercise 6.31) Suppose $p_1, .., p_k \in [1, \infty]$ satisfy $p_1^{-1} + ... + p_n^{-1} \leq 1$. Let

$$r := (p_1^{-1} + ... + p_n^{-1})^{-1}.$$

Show that if $f_i \in L^{p_i}(\mu)$ for $1 \leq i \leq n$ then $f_1 \cdot \dots \cdot f_n \in L^r(\mu)$ and

$$||f_1 \cdot \dots \cdot f_n||_r \leq ||f_1||_{p_1} \cdot \dots \cdot ||f_n||_{p_n}.$$

3. (Folland, Exercise 6.41) Let $p \in (1, \infty]$ and let $q = \frac{p}{p-1}$. Show that if T is a continuous operator on $L^p(\mathbb{R}^n)$ such that

$$\int (Tf)g = \int f(Tg)$$

for all $f, g \in L^p(\mathbb{R}^n) \cap L^q(\mathbb{R}^n)$ then T extends to a continuous operator from $L^r(\mathbb{R}^n)$ to $L^r(\mathbb{R}^n)$ for all $r \in [\min(p,q), \max(p,q)]$.