MAT 513 Midterm

March 9, 2022

Name: ______________________ **ID:** __________________

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There are **??** problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** If you use any "alternative facts" as part of your answer for any question, I reserve the right to give you an "alternative grade".

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

You have 90 minutes to complete this exam.

$\boxed{\text{12 points}}$ 1. Let $x = 0.12\,047\,\overline{047}\cdots$. Write the number x as a fraction (in lowest terms) or as a sum of two fractions in lowest terms (your choice; I prefer the latter because I am bad at arithmetic). Please fully explain your answer.

$\boxed{\text{12 points}}$ 2. State the Nested Intervals Theorem (or Nested Intervals Property, it is the same thing). Also, write one or two sentences about why it is relevant for secondary mathematics, even if never explicitly stated in this way.

12 points

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence which converges to zero, let $\{b_n\}_{n=1}^{\infty}$ be another sequence, and let $c_n = a_n \cdot b_n$ for every $n \in \mathbb{N}$

   (a) Prove or disprove: If $\{b_n\}$ is bounded, then the sequence $\{c_n\}$ converges.

   (b) Prove or disprove: If $\{b_n\}$ is monotone, then the sequence $\{c_n\}$ converges.

$\boxed{\text{12 points}}$ 4. For each of the following infinite series, decide whether it converges or diverges. Justify your answer fully.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 - n}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2 - n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} b_n$ where $b_1 = 1$ and $b_{n+1} = \dfrac{b_n}{\arctan(n)}$ for all $n \in \mathbb{N}$.

$\boxed{\text{12 points}}$ 5. Let A and B be nonempty, bounded subsets of $\mathbb{R}$. Define the set

$$A+B=\{\,a+b \mid a\in A, b\in B\,\}\ .$$

Prove that $\quad \sup A+\sup B=\sup(A+B)$.