Homework 9 (due 4/18)

MAT 342: Applied Complex Analysis

Read Sections 60–68 from Chapter 5.

Problem 1. Using the definition of the limit of a sequence, show that

$$\lim_{n\to\infty}\left(-1+i+\frac{e^{i\pi n/4}}{\sqrt{n}}\right)=-1+i.$$

Problem 2. Suppose that f is continuous in a closed bounded region R and it is analytic, non-constant and non-zero in the interior of R. Then prove that the minimum value of $|f(z)|$ in R occurs somewhere on the boundary of R and never in the interior. *Hint: Apply the Maximum Principle to the function $g(z) = 1/f(z)$ (why can it be applied?)*

Problem 3. Find the Taylor expansion of $f(z) = e^z$ around the point $z_0 = 2$ in two ways:

(i) Using Taylor's Theorem and computing $f^{(n)}(2)$, $n = 0, 1, \ldots$

(ii) Observing that $e^z = e^2 \cdot e^{z-2}$, and then using the Maclaurin expansion of e^z, with z replaced by $z - 2$.

Problem 4. Show that for $0 < |z| < 4$ we have

$$\frac{1}{4z-z^2}=\frac{1}{4z}+\sum_{n=0}^{\infty}\frac{z^n}{4^{n+2}}.$$

Hint: Use the geometric series.

Problem 5. Consider the function

$$f(z)=\frac{1}{(z-1)(z-3)}.$$

(i) Find numbers A, B such that

$$\frac{1}{(z-1)(z-3)} = \frac{A}{z-1} + \frac{B}{z-3}.$$

(ii) Write the Laurent series for $f(z)$ when $1 < |z| < 3$.

Hint: Write $\frac{1}{z-1}$ *in terms of* $\frac{1}{1-1/z}$ *and write* $\frac{1}{z-3}$ *in terms of* $\frac{1}{1-z/3}$. *Then use the geometric series.*

(iii) Write the Laurent series for $f(z)$ when $0 < |z-1| < 2$.

Problem 6. Give one Taylor series expansion and two Laurent series expansions of the function $f(z) = 1/z$ (a total of three distinct expansions). Specify the regions where the expansions are valid.