Homework 8 (due 4/11)

MAT 342: Applied Complex Analysis

Read Sections 54–59 from Chapter 4 and 60–61 from Chapter 5.

Problem 1. Let C be the positively oriented rectangle with sides parallel to the lines $x = \pm 3$ and $y = \pm 5$. Evaluate each of the following integrals:

(i) $\int_C \frac{e^{-z} \sin z dz}{(z - i\pi)^3}$ (ii) $\int_C \frac{\cos z dz}{z(2z^2 - 31i)}$

Problem 2. Compute

$$\int_C \frac{1}{(z^2+4)^2} dz$$

where C is the positively oriented circle of radius 2 around the point i.

Problem 3. Let C be the circle $|z| = 3$, positively oriented. Consider the function

$$g(z) = \int_C \frac{2w^2 - w - 2}{w - z} dw$$

defined for $|z| \neq 3$. Show that $g(2) = 8\pi i$ and compute $g(z)$ when $|z| > 3$.

Problem 4. Show that if f is analytic within and on a simple closed contour C, and z_0 is not on C, then

$$\int_C \frac{f'(z)dz}{z - z_0} = \int_C \frac{f(z)dz}{(z - z_0)^2}.$$

Problem 5. Suppose that f is an entire function, and consider $u(z) = \operatorname{Re} f(z)$. Assume that u is bounded above, so there exists a constant $c > 0$ such that $u(z) \leq c$ for all $z \in \mathbb{C}$. Show that the functions f and u must be constant in $\mathbb{C}$. *Hint: Apply Liouville's theorem to the function* $g(z) = e^{f(z)}$.

Problem 6. Is the following statement true?
Suppose that f is continuous in a closed bounded region R and it is analytic and non-constant in the interior of R. Then the minimum value of $|f(z)|$ in R occurs somewhere on the boundary of R and never in the interior.