Homework 3 (due 2/21)

MAT 342: Applied Complex Analysis

Read Sections 19–26 from Chapter 2.

Problems from the textbook:
§20: 1, 2, 8(b), 9
§24: 1(c)(d), 2(b)(d), 3(b)(c), 4

Additional problems to hand in:

Problem 1. Define

$$f(z) = \begin{cases} \frac{\overline{z}^2}{z} & \text{when } z \neq 0, \\ 0 & \text{when } z = 0. \end{cases}$$

We know from Problem 9, §20, that $f'(0)$ does not exist, so f is not differentiable at 0. Nevertheless, the Cauchy-Riemann equations hold at the point 0, as we proved in the lecture. Explain why we cannot apply the Theorem of §23 that gives sufficient conditions for differentiability. Provide complete justifications.

Problem 2. Consider the sector

$$S = \{z \in \mathbb{C} : 0 \leq \operatorname{Arg}(z) \leq \pi/4 \text{ and } |z| \leq 1/2\}.$$

(a) What is the boundary ∂S of S? Is the set S open, closed, or neither?

(b) Is S connected? If yes, is it a domain?

(c) Find the image of S under each of the functions $f(z) = z^2$, $g(z) = z^3$, $h(z) = z^4$. Describe the images as sets (in a similar way as S is defined) and then sketch them. *Hint: Express the functions in polar coordinates.*