MAT 341 – Applied Real Analysis
Fall 2015

Midterm 1 – October 1, 2015

NAME: ______________________________

Please turn off your cell phone and put it away. You are **NOT** allowed to use a calculator.

Please show your work! To receive full credit, you must explain your reasoning and neatly write the steps which led you to your final answer. If you need extra space, you can use the other side of each page.

Academic integrity is expected of all students of Stony Brook University at all times, whether in the presence or absence of members of the faculty.

PROBLEM	SCORE
1	
2	
3	
4	
TOTAL	

Problem 1: (25 points) Consider the function

$$f(x) = \begin{cases} -x & \text{if} \quad -2 \le x < 0 \\ x & \text{if} \quad 0 \le x < 2, \end{cases} \qquad f(x+4) = f(x).$$

Find the Fourier series for f. Determine whether the series converges uniformly or not. To what value does the Fourier series converge at $x = 2015$?

Problem 2: (25 points) Suppose that the Fourier series of $f(x)$ is $f(x) = \sum_{n=1}^{\infty} e^{-341n} \cos(n\pi x)$.

a) What is the Fourier series of $1 - 2f(x)$?

b) What is the Fourier series of $F(x) = \int_0^x f(y)\, dy$?

c) Find the Fourier series of $f''(x)$ if it exists. Otherwise, explain why it does not exist.

d) What is the period of f? Can f have jump discontinuities or is it a continuous function?

Problem 3: (25 points) Consider the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - S\frac{\partial u}{\partial x} = \frac{1}{k}\frac{\partial u}{\partial t}, \qquad 0 < x < 2, \quad t > 0$$

with boundary conditions

$$u(0,t) = T_0, \quad u(2,t) = 0, \qquad t > 0$$

and initial condition $u(x,0) = f(x)$, $0 \leq x \leq 2$. (S and T_0 are positive constants.)

a) Find the steady-state solution $v(x)$. What is the ODE that $v(x)$ satisfies?

b) State the initial value–boundary value problem satisfied by the transient solution $w(x,t)$. You are NOT asked to solve this problem.

Problem 4: (25 points) Solve the heat problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{4}\frac{\partial u}{\partial t}, \qquad 0 < x < 1, \quad t > 0;$$

$$u(0,t) = 0, \quad u(1,t) = \beta, \quad t > 0;$$

$$u(x,0) = \beta x + \sin\left(\frac{\pi x}{2}\right), \quad 0 \le x \le 1.$$

Some useful formulas & trigonometric identities:

$$\int x\cos(ax)\,dx = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a} + C$$

$$\int x\sin(ax)\,dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a} + C$$

$$\sin(ax)\sin(bx) = \frac{\cos((a-b)x) - \cos((a+b)x)}{2}$$

$$\sin(ax)\cos(bx) = \frac{\sin((a-b)x) + \sin((a+b)x)}{2}$$

$$\cos(ax)\cos(bx) = \frac{\cos((a-b)x) + \cos((a+b)x)}{2}$$

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

$$\sin^2(a) = \frac{1-\cos(2a)}{2} \qquad \cos^2(a) = \frac{1+\cos(2a)}{2}$$