Sample Final, MAT 324, 2011

Final will be Tuesday 12/13/2011 02:15-04:45 Room 4-130, Math

Answer each question on the paper provided. Write neatly and give complete answers. Each question is worth 10 points.

1. Define: null set, outer measure, Lebesgue measureable set.

2. State: Fatou's lemma, the monotone convergence theorem, the dominated convergence theorem, the Beppo-Levi theorem.

3. Show that it is impossible to define an inner product on the space $L^1([0,1])$ with the norm $\|\cdot\|_1$.

4. Given an example of a sequence $\{f_n\}$ on $[0,1]$ which converges to the zero function in L^1 but does not converge to it pointwise at any point.

5. If f is a measureable function on $[0,1]$, show that $\|f\|_\infty = \lim_{p\to\infty} \|f\|_p$.

6. State Hölder's inequality.

7. State Fubini's theorem. Give an example to show it can fail if the function is measurable, but not integrable.

8. State the Radon-Nikodym theorem.

9. Let ν and μ be finite Borel measures on $[0,1]$. Prove that $\nu \ll \mu$ if and only if for every $\epsilon > 0$ there is a $\delta > 0$ so that for every Borel set F, $\mu(f) < \delta$ implies $\nu(F) < \epsilon$.

10. Define absolute continuity and bounded variation. Prove that any absolutely continuous function on $[0,1]$ is of bounded variation.