MAT 324: Real Analysis, Fall 2017
Basic Review Questions I

1. Example of monotone class on $\mathbb{Z}^+$, which is not a σ-field:

2. Example of field on $\mathbb{Z}^+$, which is not a σ-field:

3. Example of σ-field on $\mathbb{Z}^+$:

   (a)

   (b)

   (c)

4. Another example of σ-field on $\mathbb{Z}^+$:

   (a)

   (b)

   (c)

5. A third example of σ-field on $\mathbb{Z}^+$:

   (a)

   (b)

   (c)

   Leave (a)-(c) above blank at first and do not continue below right away

6. Definition of measure $\mu\colon \mathcal{F}\longrightarrow \mathbb{R}^{\geq 0}$ on measurable space $(X, \mathcal{F})$ (2 properties):

7. Deduce other properties of μ from the above two (finite additivity, monotonicity, finite and countably subadditivity, continuity from below (for unions of increasing sequences of sets), continuity from above (for intersections of decreasing sequences of sets)):

8. Example of necessity of the extra condition for continuity from above:

8. Give two distinct, independent conditions sufficient for $\int_{(a,b)} f \mathrm{d}m = \int_a^b f \mathrm{d}x$:

    (a)

    (b)

9. Give two distinct examples illustrating the need for the above conditions:

    (a)

    (b)

10. Give two distinct, independent conditions sufficient for $\lim_{n \to \infty} \left(\int_X f_n \mathrm{d}m \right) = \int_X \left(\lim_{n \to \infty} f_n \right) \mathrm{d}m$:

    (a)

    (b)

11. Give two distinct examples illustrating the need for the above conditions:

    (a)

    (b)

12. Give two distinct, independent conditions sufficient for $\int_a^b \left(\int_c^d f \mathrm{d}y \right) \mathrm{d}x = \int_c^d \left(\int_a^b f \mathrm{d}x \right) \mathrm{d}y$:

    (a)

    (b)

13. Give two distinct examples illustrating the need for the above conditions:

    (a)

    (b)