MAT 324, Fall 2012
Review for final

Things to know and do for the final exam:
Everything from before the midterm; e. g.:

1. Finite, countable and uncountable sets
2. The power set of a set. Prove that the number of elements in the power set of X is greater than the number of elements in X. Corollary: the set of real numbers is uncountable.
3. Prove that there are countably many rational numbers.
4. Let $f : X \to Y$. and let $S \subset Y$. Define $f^{-1}(S)$.
5. Define an equivalence relation and an equivalence class. Let x and y be real numbers and define $x \sim y$ iff $x - y$ is rational. Is this an equivalence relation?
6. Indicator or characteristic function,
7. Define open sets of the reals and also closed sets.
8. Define a sigma field and Borel sets.
9. Understand the least upper bound property of the reals
10. Balls, open sets and rectangles in $\mathbf{R}^n$.
11. Define the Riemann integral and understand the Riemann criterion which guarantees that the integral exists.
12. Define the sup-norm and L^2-norm of a function. Does sup-norm or pointwise convergence of functions imply convergence of their Riemann integrals?
13. Define outer measure of a subset of $\mathbf{R}$ and define a Lebesque measurable set.
14. Prove that the measure of an interval is the length of the interval.
15. Prove that a countable set has Lebesgue measure zero.
16. Define the Cantor set and show that it is uncountable and has measure zero.
17. Construct a non-measurable set.
18. Show that the set of Lebesgue measurable sets is a sigma field.
19. Define a probability measure and conditional probability.
20. Define independence of sets and sigma fields with respect to a probability measure.
21. Define Lebesque and Borel measurable functions.

22. Show that the sum and product of measurable functions is measurable with respect to Lebesgue or Borel measure.

Items from after the midterm

1. Simple functions and their integrals and definition and properties of Lebesque integral
2. Definition of essential supremum and essential infimum
3. Dirac measure
4. Fatou's lemma
5. Monotone and dominated convergence theorems for sequences of functions
6. The equivalence of functions defined by "almost everywhere"
7. The bell curve $f(x) = \frac{1}{\pi}e^{-x^2/2}$
8. Topological, metric and vector spaces and relations between them
9. Norms and inner products and the Schwartz inequality
10. The spaces $L^1(E)$ and $L^2(E)$
11. Beppo Levi theorem
12. Relations between the Riemann and Lebesgue integrals
13. A function is Riemann intgrable on an interval iff its discontinuities form a set of measure zero.
14. Improper Riemann integrals and their relation to the Lebesque integral.
15. $L^1(E)$ is complete