MAT 324 – Real Analysis
Fall 2014

Midterm – October 23, 2014

NAME: ______________________________

Please turn off your cell phone and put it away. You are **NOT** allowed to use a calculator.

Please show your work! To receive full credit, you must explain your reasoning and neatly write the steps which led you to your final answer. If you need extra space, you can use the other side of each page.

Academic integrity is expected of all students of Stony Brook University at all times, whether in the presence or absence of members of the faculty.

PROBLEM	**SCORE**
1	
2	
3	
4	
TOTAL	

Problem 1: (25 points) Let $E_1, E_2, \ldots, E_{2014}$ be measurable subsets of $[0,1]$.

a) Suppose $m(E_k) > 1 - \dfrac{1}{2^k}$ for each $1 \leq k \leq 2014$. Show that $m\left(\bigcap_{k=1}^{2014} E_k\right) > 0$.

b) Suppose almost every x from the interval $[0,1]$ belongs to at least 3 of these subsets. Prove that there exists at least one set E_k with $1 \leq k \leq 2014$ such that $m(E_k) \geq \frac{3}{2014}$.

Hint: The function $f(x) = \sum_{k=1}^{2014} \chi_{E_k}(x)$ has the property that $f(x) \geq 3$ a.e.

Problem 2: (20 points) Does there exist a Lebesgue measurable subset E of $\mathbb{R}$ such that for every interval (a, b) we have

$$m\,(E \cap (a, b)) = \frac{b-a}{2}\,?$$

Either construct such a set or prove it does not exist.

Problem 3: (25 points) Let E be a measurable set and $f : E \to \mathbb{R}$ Lebesgue integrable on E. Define $E_k = \left\{x \in E \mid |f(x)| < \frac{1}{k}\right\}$ for $k \geq 1$.

a) Show that each E_k is a measurable set.

b) Determine whether $\{E_k\}$ is an increasing or decreasing collection of sets.

c) Show that $\lim_{n\to\infty} \int_{E_k} |f| \, dm = 0$.

Problem 4: (30 points) Compute the following limit if it exists and justify the calculations. If the limit does not exist explain why it does not exist.

a) $\lim_{n\to\infty} \int_0^1 \frac{\sqrt{n}}{\sqrt{x}} \cdot \chi_{[0,\frac{1}{n}]}\, dx$

b) $\lim_{n\to\infty} \int_a^\infty \frac{n\sin(\sqrt{x})}{1+n^2x^2}\,dx,$ where $a > 0$