Midterm 2 Practice Problems

Problem 1. Verify that the functions $y_1 = x$, $y_2 = x^3$ are solutions of the differential equation

$$x^2y'' - 3xy' + 3y = 0.$$

Solve the initial value problem

$$y(-1) = 1, y'(-1) = -2.$$

Problem 2. Verify that the functions $y_1 = x^2$, $y_2 = x^{-1}$ are solutions of the differential equation

$$x^2y'' - 2y = 0.$$

Solve the initial value problem

$$y(1) = 5, y'(1) = -3.$$

Problem 3. Show that the functions

$$2 + e^2 - 3\sin x,\ 1 + 2e^2 - 3\sin x,\ e^2 - \sin x$$

are linearly dependent on the real line.

Problem 4. Using Wronskian to show that the functions $y_1 = x^2\,, y_2 = \sin x, y_3 = \cos x$ are linearly independent on the real line.

Problem 5. Solve the following initial value problems.

1. $y^{(3)} + 2y'' + y' = 0$, $y(0) = 2$, $y'(0) = -1$, $y''(0) = 0$.
2. $y^{(3)} - 3y'' + 3y' - y = 0$, $y(0) = 1$, $y'(0) = 1$, $y''(0) = 2$.

Problem 6. Find the general solution of the following equations.

1. $y^{(3)} - 8y = 0$.
2. $y^{(5)} - y' = 0$.
3. $y^{(3)} - 5y'' + 8y' - 4y = 0$.
4. $y^{(4)} + 2y'' + y = 0$.

Problem 7. Find the general solution of the following nonhomogeneous equations.

1. $y'' - y' - 2y = 3x + 4$.
2. $y'' - 4y = 2e^{2x}$.
3. $y'' + y = \sin x + x \cos x$.

Problem 8. Solve the initial value problem

$$y'' - 7y' + 6y = \sin(3x),\ y(0) = 3,\ y'(0) = 2.$$