# MAT303: Calculus IV with applications
## Practice Midterm 1
### Spring 2016

Midterm I is scheduled on March 2nd in class (Library W4550, 10am-10.53am). It will cover sections 1.1 through 1.6. of the book and it will consist of 5 problems. The use of calculators, books and notes is not allowed. Moreover every single answer you give to the problems should be well motivated so that the grader can easily understand your work. For a good preparation for the midterm you should work out and master *all* problems in this practice exam (even the problems with an asterisk). For further problems and exercises you can consult the book or past midterms of previous MAT303 classes (see the link *http://www.math.stonybrook.edu/mathematics-department-course-web-pages*).

Regarding Assignment 4, you will need to turn in *all* problems without the asterisk, plus 2 problems with an asterisk at your choice. Assignment 4 is due Monday February 29th.

**Exercise 1:** Find the general solution of the following differential equations:

(i).
$$\frac{dy}{dx} = xy - \frac{x}{y}$$

(ii).
$$\frac{dy}{dx} = \sqrt{x+y+1}$$

(iii).
$$(x+2y)\frac{dy}{dx} = y$$

(iv).
$$\frac{dy}{dx} = x - \frac{1}{x^2-2y}$$

**Exercise 2*:** Find the solution of the following initial value problems:

(i).
$$y\,\frac{dy}{dx} - y = \sqrt{x^2+y^2}; \qquad y(1) = 1$$

(ii).
$$x^2y + 2xy = 5y^4; \qquad y(1) = 0$$

**Exercise 3:** Find a general solution of the following linear first-order equations:

(i).
$$2x\frac{dy}{dx} - 3y = 9x^3$$

(ii).
$$x\frac{dy}{dx} + y = 3xy \qquad y(1) = 0$$

(iii).
$$x^3\,\frac{dy}{dx} = xy + 1; \qquad y(1) = 0$$

**Exercise 4:** Check the exactness of the following differential equations and find the solution.

(i).
$$(3x^2 + 2y^2)dx \,+\, (4xy + 6y^2)dy \,=\, 0$$
(ii).
$$\big(x \,+\, \arctan(y)\big)dx \,+\, \Big(\frac{x+y}{1+y^2}\Big)dy \,=\, 0$$
(iii).
$$\big(e^x \sin(y) \,+\, \tan(y)\big)dx \,+\, \big(e^x \cos(y) \,+\, x\sec^2(y)\big)dy \,=\, 0$$

**Exercise 5:** Check whether the hypothesis of the theorem of existence and uniqueness for initial value problems are satisfied for the following problem. If so, find the unique solution.

(i).
$$\frac{dy}{dx} \,=\, \frac{x-1}{y}; \qquad y(1) \,=\, 0$$
(ii).
$$\frac{dy}{dx} \,=\, \frac{x-1}{y}; \qquad y(0) \,=\, 1$$
(iii).
$$\frac{dy}{dx} \,=\, \sqrt[3]{y}; \qquad y(-1) \,=\, -1$$
(iv).
$$\frac{dy}{dx} \,=\, \sqrt{y}; \qquad y(-1) \,=\, -1.$$
(v). In general, for which $y_0$ does the initial value problem
$$\frac{dy}{dx} \,=\, \sqrt{y}; \qquad y(0) \,=\, y_0$$
admit a unique solution?

**Exercise 6:** A car travelling at 88 ft/s skids 176 ft after its brakes are suddenly applied. Under the assumption that the braking system provides constant deceleration, what is that deceleration?. For how long does the skid continue?

**Exercise 7:** The amount $A(t)$ of atmospheric pollutants in a certain mountain valley grows naturally (i.e. it grows according to the exponential equation $\frac{dA}{dt} = kA$ where $k$ is a constant) and is tripling every 7.5 years. If the initial amount is 10 pu (pollutant units), write a formula for $A(t)$ giving the amount (in pu) present after $t$ years. How long does it take for $A(t)$ to centuplicate?

**Exercise 8*:** A tank initially contains 60 gal of pure water. Brine containing 1 lb of salt per gallon enters the tank at 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min; thus the tank is empty after exactly 1 h. **(a)** Find the amount of salt in the tank after $t$ minutes. **(b)** What is the maximum amount of salt ever in the tank?

**Exercise 9* (Newton's law of cooling):** According to Newton's law of cooling, the time rate of change of the temperature $T(t)$ of a body immersed in a medium of constant temperature $A$ is proportional to the difference $A - T$. That is,
$$\frac{dT}{dt} \,=\, k\,(A \,-\, T)$$
where $k$ is a positive constant. Use now these information to answer to the following problem.

A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F. After 30 min the temperature of the cake is 140°F. When will it be 100°F?

**Exercise 10*:** By using partial fraction integration technique, find the solution of the following initial value problem:

$$\frac{dP}{dt} = \frac{kP(10-P)}{10}; \quad P(0) = 1$$

where $k > 0$ is a positive constant (you can assume that both $t$ and $P$ are positive variables). When $t$ gets large, does $P(t)$ approach some fixed value? If yes, what is this value?