1.

Part II

Problem 5

$$x^2 y'' + x y' - y = 72 x^5$$

Verify that $y_c = c_1 x + \frac{c_2}{x}$ is a compliment ary solution.

$y_c' = c_1 - \frac{c_2}{x^2}$ $\quad$ $y_c'' = \frac{2 c_2}{x^3}$ $\quad$ → Plug into the equation.

$$x^2 \left(\frac{2 c_2}{x^3} \right) + x \left(c_1 - \frac{c_2}{x^2} \right) - c_1 x - \frac{c_2}{x} =$$

$$= \frac{2 c_2}{x} + c_1 x - \frac{c_2}{x} - c_1 x - \frac{c_2}{x} = 0$$

$\boxed{\text{OK}}$

Variation of parameters

$y_1 = x \qquad y_2 = \frac{1}{x} \qquad W = \det \begin{pmatrix} x & 1/x \\ 1 & -1/x^2 \end{pmatrix} = -\frac{2}{x}$

$\cancel{Y_p(x) = -Y_1 \int \frac{Y_2 f}{W} dx + Y_2 \int \frac{Y_1 f}{W} dx}$

[illegible]

Standard form $\quad y'' + \frac{y'}{x} - \frac{1}{x^2} y = 72 x^3$ $\quad$ $f(x)$.

$$Y_p(x) = - Y_1 \int \frac{Y_2 f}{W} dx + Y_2 \int \frac{Y_1 f}{W} dx =$$

$$= -x \int \frac{72x^2}{-2/x}\,dx + \frac{1}{x}\int \frac{72x^4}{-2/x}\,dx$$

$$= -x \int -36x^3\,dx + \frac{1}{x}\int -36x^5\,dx$$

$$= 36x\left(\frac{x^4}{4}\right) + \frac{1}{x}(-36)\frac{x^6}{6} =$$

$$= 9x^5 - 6x^5 = \boxed{3x^5}$$

Problem 6

$$25x'' + 10x' + 226x = 0$$

$$x(0) = 20$$

$$x'(0) = 41.$$

$$25r^2 + 10r + 226 = 0$$

$$r = -5 \pm \sqrt{25 - (25)(226)} =$$

$$= -5 \pm \sqrt{25(-225)} =$$

$$= -5 \pm 5\sqrt{-225} = -5 \pm 5(15)i$$

$$= -5 \pm 75i$$

③

$$x(t) = A e^{-5t} \cos(75t) + B e^{-5t} \sin(75t)$$

Impose initial conditions:

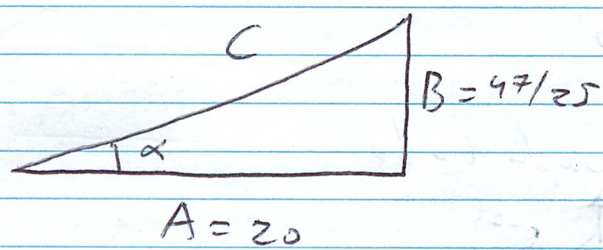
$$x'(t) = -5 A e^{-5t} \cos(75t) - A e^{-5t} \sin(75t)(75)$$
$$-5B e^{-5t} \sin(75t) + B e^{-5t} \cos(75t)(75)$$

$$\begin{cases} 20 = x(0) = A \\ 41 = x'(0) = -5A + 75B \end{cases}$$

$A = 20$ $\qquad B = \dfrac{5A + 41}{75} = \dfrac{100 + 41}{75} = \dfrac{141}{75} = \dfrac{47}{25}$

$$x(t) = 20 e^{-5t} \cos(75t) + \frac{47}{25} \sin(75t)$$

Standard form



$$C = \sqrt{400 + \frac{2209}{625}} = \sqrt{\frac{252209}{625}} = \frac{\sqrt{252209}}{25}$$

~~Sin~~ As both $A > 0$, $B > 0$ are positive, α is simply given by $\alpha = \tan^{-1}(B/A) =$

$$= \tan^{-1}\left(\frac{47/25}{20}\right) =$$

$$= \tan^{-1}\left(\frac{47}{500}\right) = 0.094 \text{ rad.}$$

The standard form is given by

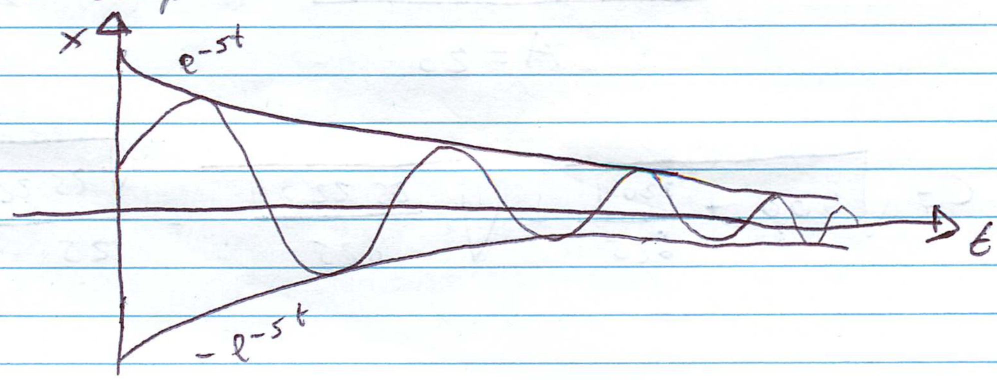
$$x(t) = C e^{-5t}\left(\cos(75t - 0.049)\right)$$

$$= \frac{\sqrt{252209}}{25} e^{-5t} \cos(75t - 0.049)$$

$\omega_0 = 75$ circular frequency.

$\alpha = 0.049$ phase

$\frac{\sqrt{252209}}{25} e^{-5t}$ = amplitube



5

Problem 7

a

b $\begin{cases} x'' = -4x + \sin t \\ y'' = 4x - 8y \end{cases}$

$$\begin{cases} x'' + 4x = \sin t \\ y'' + 8y - 4x = 0 \end{cases} \rightarrow \begin{cases} (D^2+4)x = \sin t \\ -4x + (D^2+8)y = 0 \end{cases}$$

Eliminate x

Multiply R_1 by 4 and R_2 by (D^2+4). Then sum up the rows.

$$\begin{cases} 4(D^2+4)x = 4\sin t \\ -4(D^2+4)x + (D^2+4)(D^2+8) = 0 \end{cases}$$

$$(D^2+4)(D^2+8)y = 4\sin t$$

$$y^{(iv)} + 12y'' + 32y = 4\sin t$$

$$y(x) = y_c + y_p$$

Find y_c: $(r^2+4)(r^2+8) = 0$

$$r = \pm 2i \qquad r = \pm\sqrt{8}\,i$$

$$Y_c = c_1\cos(2t) + c_2\sin(2t) + c_3\cos(\sqrt{8}\,t) + c_4\sin(\sqrt{8}\,t)$$

Find y_p

Guess: $y_p = A\cos t + B\sin t$

$y_p' = -A\sin t + B\cos t$

$y_p'' = -A\cos t - B\sin t$

$y_p''' = A\sin t - B\cos t$

$y_p^{(iv)} = A\cos t + B\sin t$

Plug-in.

$A\cos t + B\sin t - 12A\cos t - 12B\sin t$

$+ 32A\cos t + 32B\sin t = 4\sin t$

$\cos t(21A) + \sin t(21B) = 4\sin t$

$\begin{cases} 21A = 0 \\ 21B = 4 \end{cases} \to \boxed{\begin{matrix} A = 0 \\ B = 4/21 \end{matrix}}$

$Y = c_1\cos(2t) + c_2\sin(2t) + c_3\cos(\sqrt{8}\,t) + c_4\sin(\sqrt{8}\,t)$

$+ 4/21\sin t.$

Now find $x(t)$:

From the second equation $-4x + (D^2+8)y = 0$

we find $x = \frac{(D^2+8)y}{4} = y''/4 + 2y = \ldots$

c

$$\begin{cases} (D^2+1)x + D^2 y = 2e^{-t} \\ (D^2-1)x + D^2 y = 0 \end{cases}$$

Eliminate y: subtract the 2 rows.

$2x = 2e^{-t} \longrightarrow \boxed{x = e^{-t}}$

Plug $x = e^{-t}$ in the second row.

$(D^2-1)e^{-t} + D^2 y = 0$

$(e^{-t})'' - e^{-t} + y'' = 0$

$e^{-t} - e^{-t} + y'' = 0$

$y'' = 0$

$r^2 = 0 \longrightarrow r = 0$ multipl. 2

$\boxed{y(t)} = c_1 e^{0t} + c_2 t e^{0t} = \boxed{c_1 + c_2 t}$

Problem 8

$$\begin{cases} x_1' = x_1 + x_4 \\ x_2' = -x_2 - 2x_3 \\ x_3' = 2x_2 - x_3 \\ x_4' = x_4 \end{cases}$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{= A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 0 & 0 & 1 \\ 0 & -1-\lambda & -2 & 0 \\ 0 & 2 & -3-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{pmatrix}$$

$$= -(1-\lambda) \det \begin{pmatrix} 1-\lambda & 0 & 0 \\ 1 & -1-\lambda & -2 \\ 0 & 2 & -3-\lambda \end{pmatrix}$$

$$= -(1-\lambda)(1-\lambda) \det \begin{pmatrix} -1-\lambda & -2 \\ 2 & -3-\lambda \end{pmatrix} =$$

Problem 8

$$\begin{cases} x_1' = x_1 + x_4 \\ x_2' = -x_2 - 2x_3 \\ x_3' = 2x_2 - x_3 \\ x_4' = x_4 \end{cases}$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\shortparallel \atop A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 0 & 0 & 1 \\ 0 & -1-\lambda & -2 & 0 \\ 0 & 2 & -1-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{pmatrix}$$

$$= (1-\lambda) \det \begin{pmatrix} -1-\lambda & -2 & 0 \\ 2 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{pmatrix}$$

$$= (1-\lambda)(1-\lambda) \det \begin{pmatrix} -1-\lambda & -2 \\ 2 & -1-\lambda \end{pmatrix}$$

$$= (1-\lambda)^2 \left((-1-\lambda)^2 + 4 \right) = (1-\lambda)^2 \left(\lambda^2 + 2\lambda + 5 \right) = 0$$

$$\boxed{\lambda = 1 \quad \text{multiple} \quad 2}$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$\boxed{\lambda = -1 \pm \sqrt{1-5} = -1 \pm 2i}$$

<u>Eigenvectors for $\lambda = -1 \pm 2i$</u>

Plug $\lambda = -1 + 2i$

Look for $\underline{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

solutions of

$$\begin{pmatrix} 2-2i & 0 & 0 & 1 \\ 0 & 1-2i & -2 & 0 \\ 0 & 2 & 1-2i & 0 \\ 0 & 0 & 0 & 2-2i \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} (2-2i)a + d = 0 & \rightarrow a = 0 \\ (1-2i)b - 2c = 0 & \\ 2b + (1-2i)c = 0 & \rightarrow b = \frac{-(1-2i)c}{2} \\ (2-2i)d = 0 & \rightarrow d = 0 \end{cases}$$

Hence c is free

(11)

Can take $c = 1$.

Hence $b = -\frac{1}{2} + i$

An eigenvector is $\underline{v} = \begin{pmatrix} 0 \\ -1/2 + i \\ 1 \\ 0 \end{pmatrix}$

The solution associated to it is

$$\underline{x}(t) = \underline{v}\, e^{\lambda t} = \begin{pmatrix} 0 \\ -1/2 + i \\ 1 \\ 0 \end{pmatrix} e^{(-1+2i)t} =$$

$$= \begin{pmatrix} 0 \\ -1/2 + i \\ 1 \\ 0 \end{pmatrix} e^{-t} \left(\cos(2t) + i \sin(2t) \right)$$

From this we deduce 2 linearly indep. **real** solutions by taking the real and imaginary parts of $\underline{x}(t)$.

$$\boxed{\begin{aligned} \underline{x}_1(t) &= \mathrm{Re}\, \underline{x} = \begin{pmatrix} 0 \\ -1/2 \cos(2t) - \sin(2t) \\ \cos(2t) \\ 0 \end{pmatrix} e^{-t} \\ \underline{x}_2(t) &= \mathrm{Im}\, \underline{x} = \begin{pmatrix} 0 \\ -1/2 \sin(2t) + \cos(2t) \\ \sin(2t) \\ 0 \end{pmatrix} e^{-t} \end{aligned}}$$

Eigenvectors for $\lambda=1$

Plug $\lambda = 1$

Look for $\underline{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ~~st~~ such that

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} d = 0 \\ -2b - 2c = 0 \to b = -c \\ 2b - 2c = 0 \to b = c \end{cases} \to c = -c \to c = 0 \to b = 0$$

Hence a is free.

We get one eigenvector $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

But $\lambda = 1$ has multiplicity 2, so we want 2 eigenvectors.

We start over ~~by~~ by looking for generalized eigenvectors.

The defect is $2 - 1 = 1$,

Hence we look for vectors $\underline{v}_2 \neq 0$ such that

$$(A - I)^2 \underline{v}_2 = 0$$

and $(A - I)\underline{v}_2 \stackrel{\cdot}{=} \underline{v}_1 \neq 0$.

$$(A-I)^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{cases} 8c = 0 \\ 8b = 0 \end{cases} \rightarrow \boxed{\begin{matrix} c = 0 \\ b = 0 \end{matrix}}$ Hence a, d are free.

We get a vector by $\begin{matrix} a = 1 \\ d = 0 \end{matrix} \rightarrow \underline{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

~~Check:~~

$$\cancel{(A-I)\,\underline{v}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}$$

~~**No!** $\underline{v}_1$ we chose is not good.~~

~~We choose another $\underline{v}_1$ by $a=0$, $d=1$~~

$$\cancel{\underline{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}$$

Check:

$$(A-I)\,\underline{v}_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

No! The vector $\underline{v}_2$ we chose does not work.
Hence we choose another $\underline{v}_2$ by setting $a=0$, $d=1$. So we have $\underline{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Check:

$$(A-I)\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\underline{v}_1} \neq \underline{0}$$

Therefore we have found 2 generalized eigenvectors.

$$\underline{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \underline{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The 2 linearly indep. solutions are:

$$\underline{x}_3(t) = \underline{v}_1 e^{\lambda t} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^t$$

$$\underline{x}_4(t) = \left(\underline{v}_1 t + \underline{v}_2\right) e^{\lambda t} = \left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} t + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right) e^t$$

$$= \begin{pmatrix} t \\ 0 \\ 0 \\ 1 \end{pmatrix} e^t$$

Putting everything together, the general solution of the system is given by

$$\boxed{\underline{x}(t) = c_1 \underline{x}_1(t) + c_2 \underline{x}_2(t) + c_3 \underline{x}_3(t) + c_4 \underline{x}_4(t).}$$

(b)

(c)
$$\begin{cases} x_1' = x_1 \\ x_2' = x_1 + 3x_2 + x_3 \\ x_3' = -2x_1 - 4x_2 - x_3 \end{cases}$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & 1 \\ -2 & -4 & -1 \end{pmatrix}}_{\overset{\shortparallel}{A}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\det(A-\lambda I) = \det\begin{pmatrix} 1-\lambda & 0 & 0 \\ 1 & 3-\lambda & 1 \\ -2 & -4 & -1-\lambda \end{pmatrix} =$$

$$= (1-\lambda)\det\begin{pmatrix} 3-\lambda & 1 \\ -4 & -1-\lambda \end{pmatrix} =$$

$$= (1-\lambda)\Big((3-\lambda)(-1-\lambda)+4\Big) =$$

$$= (1-\lambda)\left(-3-3\lambda+\lambda+\lambda^2+4\right) =$$

$$= (1-\lambda)(\lambda^2-2\lambda+1) = (1-\lambda)(\lambda-1)^2$$

$$= -(\lambda-1)^3 = 0 \longrightarrow$$

$\lambda = 1$ wt. multiplicity 3.

Hence we expect 3 eigenvectors from $\lambda = 1$.

Plug $\lambda = 1$. Look for $\underline{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ such that:

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & -4 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$a + 2b + c = 0$$

$$\cancel{-2a - 4b - 2c = 0}$$

$a = -2b - c$ Hence b and c are free

We obtain 2 eigenvectors,

for instance $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

But they are not enough.

$\lambda = 1$ has defect $3 - 2 = 1$.

We use the technique of generalized eigenvectors.

Look for non-zero vectors $\underline{v}_2$ such that

$$(A-I)^2\,\underline{v}_2 = 0.$$

$$(A-I)^2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & -4 & -2 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & -4 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Hence any non-zero vector is solution.

We choose for instance $\underline{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

<u>Check</u>: $(A-I)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & -4 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}}_{= \underline{v}_1} \neq 0$

We got 2 generalized eigenvectors from the solution $\underline{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

~~To find the third generalized eigenvector we take another solution of $(A-I)^2\underline{v}_3 = 0$ such that $\underline{v}_1, \underline{v}_2, \underline{v}_3$ are linearly indep.~~

To find the third eigenvector $\underline{v}_3$, we take a linear combination of the original eigenvectors $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ associated to $\lambda = 1$ which ~~is~~ such that it is linearly independ. with $\underline{v}_1 = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ and $\underline{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

For instance we can take $\underline{v}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

In fact $\underline{v}_1, \underline{v}_2, \underline{v}_3$ are linearly indep. as $\det \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} = -1 \neq 0$.

The 3 linearly indep. solutions are given by,

$$\underline{x}_1(t) = \underline{v}_1 e^{\lambda t} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} e^t$$

$$\underline{x}_2(t) = (\underline{v}_1 t + \underline{v}_2) e^{\lambda t} = \begin{pmatrix} 1 \\ t \\ -2t \end{pmatrix} e^t$$

$$\underline{x}_3(t) = \underline{v}_3 e^{\lambda t} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} e^t$$

The general solution is:

$\underline{x}(t) = c_1 \underline{x}_1 + c_2 \underline{x}_2 + c_3 \underline{x}_3 =$

$$= \begin{pmatrix} c_2 - c_3 \\ c_1 + c_2 t \\ -2c_1 - 2c_2 t + c_3 \end{pmatrix} e^t$$