MAT303: Calculus IV with applications
Practice Final Exam
Spring 2016

By no means this practice exam is going to be similar to the final. For a good preparation you should work out this practice final together with the previous midterms, practice midterms and homework. Moreover, with the help of the 'summary of the course', you should be able to understand on which topics you should do more practice (look up for problems on the textbook). Remember that in mathematics even though one knows the theory, by no means one is able to solve problems. The best way to learn Calculus IV, and math in general, is to solve plenty of problems in the right way. The more problems you will do, the more you will do good in exams.

Problem 1 Find the general solution of the following differential equations.

$$(1+x)\frac{dy}{dx} = 4y$$
$$xy' = 3y + x^3$$
$$x(x+y)y' = y(x-y)$$
$$y' = (4x+y)^2$$
$$y^2y' + 2xy^3 = 6x$$
$$(1+ye^{xy})dx + (2y+xe^{xy})dy = 0$$

Problem 2 A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at the rate of 5 gal/s, and the well-mixed brine in the tank flows out at the rate of 3 gal/s. How much salt will the tank contain when it is full of brine?

Problem 3 Consider the differential equation depending on the parameter h:

$$\frac{dy}{dx} = x^3(x^2-4) - h.$$

For $h = 0$ find the critical solutions and classify them as stable or unstable. What are the bifurcation values for h?

Problem 4 Find the general solution of the following second-order differential equations.

$$y^{(iv)} + 18y'' + 81y = 0$$
$$y'' + y' + y = \sin^2(x)$$
$$y^{(3)} - y = e^x + 7$$
$$y'' + 6y' + 13y = e^{-3x}\cos(2x)$$

Problem 5 Verify that $y_c(x) = c_1x + c_2x^{-1}$ is a complementary solution for

$$x^2y'' + xy' - y = 72x^5.$$

Use variation of parameters to find a particular solution of the equation (write first the equation in standard form). You can try other similar problems as in problems 58-59 of Section 3.5.

Problem 6 Suppose that in a mass-spring-dashpot system with $m = 25$, $c = 10$, and $k = 226$ is set in motion with $x(0) = 20$ and $x'(0) = 41$. Find the position function $x(t)$ and sketch a graph of the solution.

Problem 7 Use the method of elimination to find the general solution of the following systems.

$$a)\quad x' = x + 9y; \quad y' = -2x - 5y.$$
$$b)\quad x'' = -4x + \sin(t); \quad y'' = 4x - 8y.$$
$$c)\quad (D^2+1)x + D^2 y = 2e^{-t}; \quad (D^2-1)x + D^2 y = 0.$$

Problem 8 Use the eigenvalue method to find the general solution of the following systems.

$$a)\quad x_1' = x_1 + x_4, \quad x_2' = -x_2 - 2x_3, \quad x_3' = 2x_2 - x_3, \quad x_4' = x_4.$$
$$b)\quad x_1' = -\frac{1}{5}x_1 + \frac{1}{5}x_3; \quad x_2' = \frac{1}{5}x_1 - \frac{2}{5}x_2; \quad x_3' = \frac{2}{5}x_2 - \frac{1}{5}x_3.$$
$$c)\quad x_1' = x_1, \quad x_2' = x_1 + 3x_2 + x_3, \quad x_3' = -2x_1 - 4x_2 - x_3.$$

Problem 9 Problem 29 of section 5.2 on the textbook.

Problem 10 Use the exponential matrix to find a particular solution of the following initial value problem.

$$x_1' = 5x_1, \quad x_2' = 10x_1 + 5x_2, \quad x_3' = 20x_1 + 30x_2 + 5x_3, \quad x_1(0) = 4, \quad x_2(0) = 5, \quad x_3(0) = 6.$$

Problem 11 Use the method of undetermined coefficients for linear systems to find the general solution of the following systems

$$a)\quad x' = x + 2y + 3, \quad y' = 2x + y - 3$$
$$b)\quad x' = 4x + y + e^t, \quad y' = 6x - y - e^t.$$

Problem 12 A fish population $P(t)$ in a lake has constant birth rate of 4 and a death rate equal to P (the time t is measured in days). Moreover 3 fish are harvested each day. Write a model for this problem. Find then the critical points and classify them as stable or unstable. Find the expression for $P(t)$ in function of the initial data $P(0) = P_0$ and the limiting population. Sketch the solution curves corresponding to the different values of P_0.

Problem 13 Solve other problems you can find on the textbook. Moreover look up at the problems of the previous midterms and practice midterms.