MAT 132: Calculus 2
Practice Problems for the Final

Stony Brook University Fall 2021

It is also recommended to review Practice Problems for Midterms 1 and 2.

Problem 1. Compute the following integrals:

$$\int \left(e^x + e^{-x}\right)^2 dx$$

$$\int_0^{\pi/2} 3\cos^2 x \sin x dx$$

$$\int_0^{\pi} \left(\cos^2 x + \cos^2(2x)\right) dx$$

$$\int_0^1 x^2 e^{x^3} dx$$

$$\int \frac{2-x}{x(x+1)} dx$$

$$\int \ln(x^2 + x) dx$$

Problem 2. Let R denote the region in the plane bounded by the 4 curves $x = 0$, $x = \pi$, $y = 0$, and $y = \sin x + 1$.

(a) Compute the area of R.

(b) Compute the volume when R is rotated around the x-axis.

Problem 3. A particle is moving along the x-axis; its speed at any time $t \geq 0$ is given in terms of t by the formula $t^2 e^t$.

Compute the total distance traveled by the particle during the time interval $0 \leq t \leq 2$.

Problem 4. For each of the following improper integrals, determine whether it converges or not. If the integral converges, then determine its value.

$$\int_{-1}^{2} \frac{dx}{x^3}$$

$$\int_0^{\infty} \frac{x}{x^2+1} dx$$

$$\int_0^{\infty} \frac{x}{(x^2+1)^2} dx$$

$$\int_0^{\infty} \sin^2 x dx$$

Problem 5. A spring has a natural length of 10 cm. It takes 1 J to stretch the spring from 10 cm to 15 cm. How much work would it take to stretch the spring from 5 cm to 20 cm?

Problem 6. Find the limits of the following sequences:

a) $\displaystyle \lim_{n\to\infty} \frac{3-n^2}{n^3 - n(n^2-1)},$ **b)** $\displaystyle \lim_{n\to\infty} \frac{e^{1-n}}{1+n}$

c) $\displaystyle \lim_{n\to\infty} \frac{(1+n!)^2}{(1-n!)^2},$ **d)** $\displaystyle \lim_{n\to\infty} \left(\frac{2^n}{1+2^{-n}} - 2^n\right)$

e) $\displaystyle \lim_{n\to\infty} \sqrt{\frac{n+3^n}{3^n+5}},$ **e)** $\displaystyle \lim_{n\to\infty} \frac{5n!}{2^n+1}.$

Problem 7. Determine if the following series converge absolutely, converge conditionally, or diverge. **No explanation** is required in this problem.

1) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{0.5 - 2^n}$$

☐ converges absolutely ☐ converges conditionally ☐ diverges

2) $$\sum_{n=10}^{\infty} \frac{2^n - n^5}{n!}$$

☐ converges absolutely ☐ converges conditionally ☐ diverges

3) $$\sum_{n=1}^{\infty} \frac{n^\pi + 2}{n \ln n + 1}$$

☐ converges absolutely ☐ converges conditionally ☐ diverges

4) $$\sum_{n=1}^{\infty} (-1)^n n \left(\frac{1}{n} - \frac{1}{n + \sqrt{n}} \right)$$

☐ converges absolutely ☐ converges conditionally ☐ diverges

5) $$\sum_{n=1}^{\infty} \frac{3^{-n}}{n}$$

☐ converges absolutely ☐ converges conditionally ☐ diverges

Problem 8. Consider the following Maclaurin series

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots.$$

(a) Write the Maclaurin series for $f(x) = \ln(1+2x)$ and for $g(x) = f'(x)$.
(b) What is the radius of convergence for the series in **(a)**?

Problem 9. Consider the following Maclaurin series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

(a) Write the Maclaurin series for $f(x) = x\sin(x/5)$ and for $g(x) = \int f(x)dx$.
(b) What is the radius and interval of convergence for the series in **(a)**?

Problem 10. Find the general solutions to the following differential equations

$$\frac{dy}{dt} = 2\cos(2t+1)y$$

$$x^2y' = (x+1)y$$

$$y' = e^{x+y}$$

$$y' = x^2e^y$$

Problem 10. Solve the following initial-value problems with the initial condition $y(0) =$
1

$$y' = y + 1$$

$$y' = xy$$

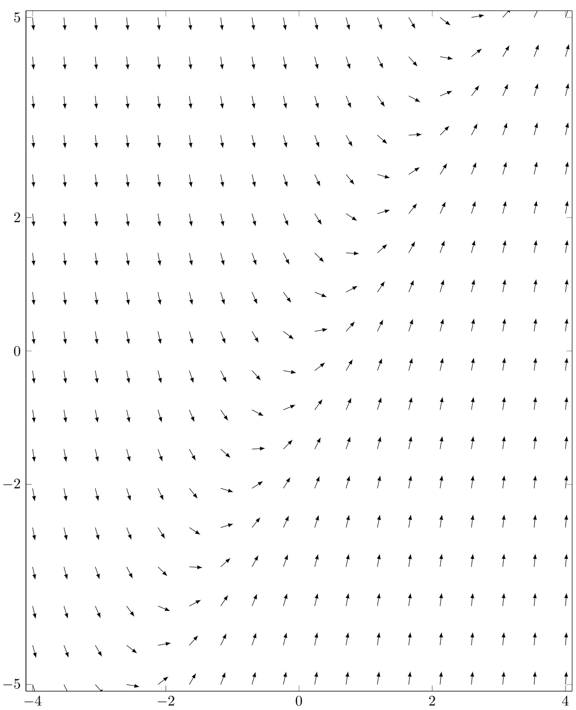
Problem 11. Match the differential equations with corresponding direction vector fields. **No explanation is required in this problem.**

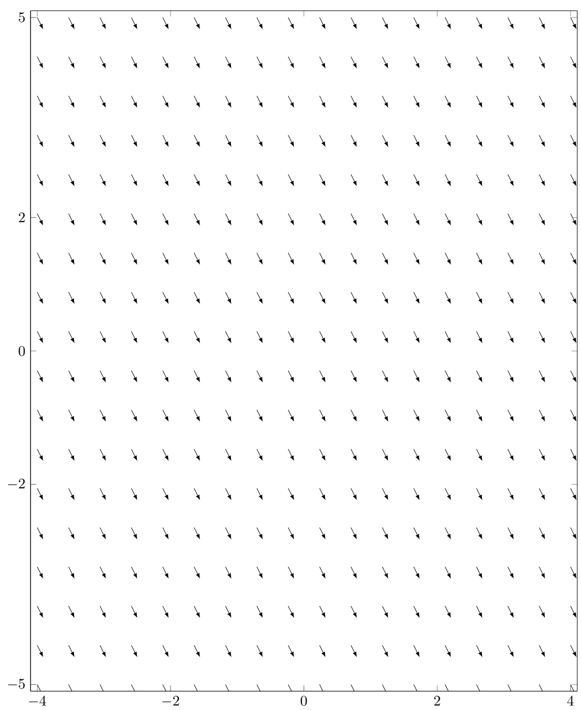
$$y' = x/y, \qquad y' = y(3-y), \qquad y' = x^2 - y^2$$

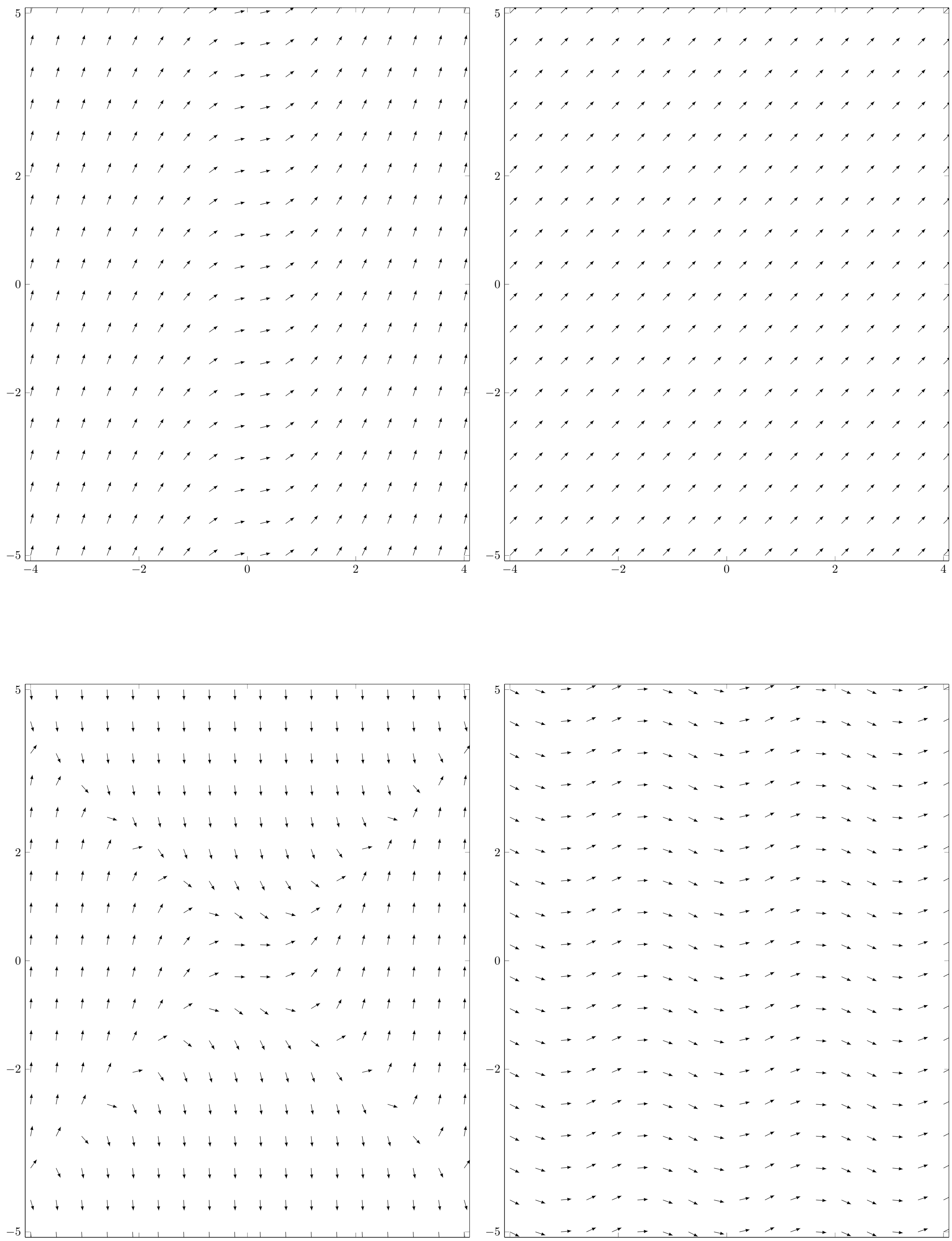
$$y' = 2x - y, \qquad y' = -2, \qquad y' = 1$$

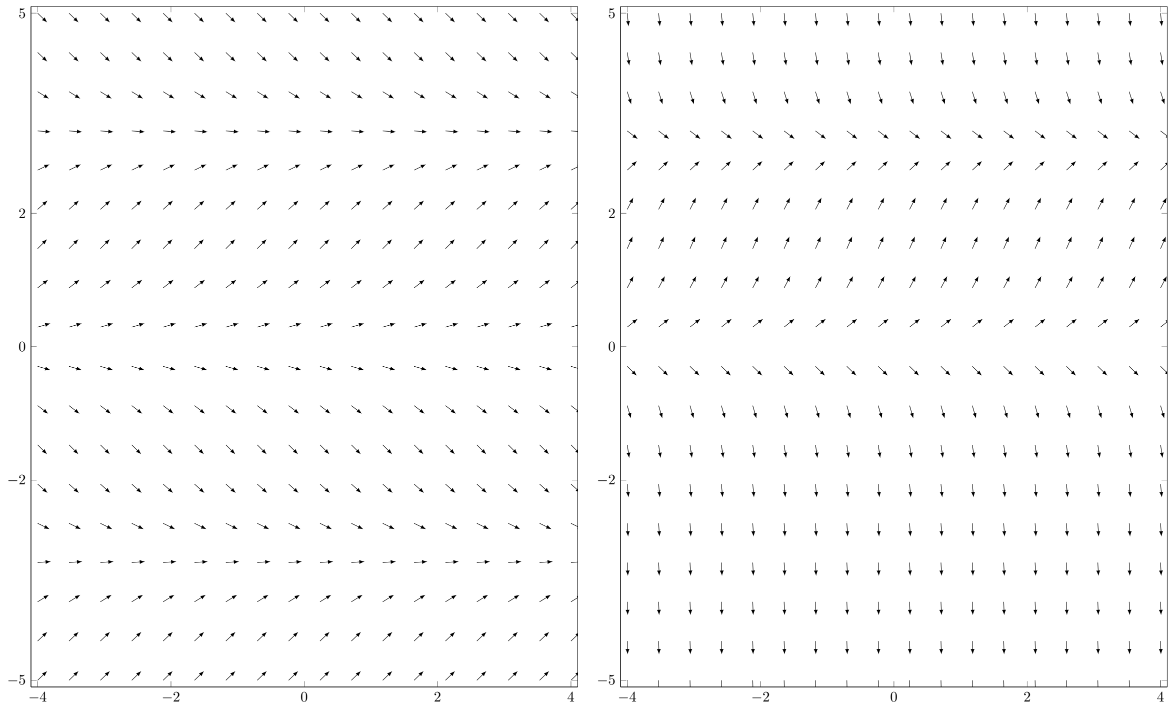
$$y' = \sin x \cos x, \qquad y' = \sin y, \qquad y' = |x|$$

(One equation is without a direction vector field.)






5
2
0
−2
−5
−4
−2
0
2
4



Problem 12. Find the general solutions to the following second order differential equations

$$y'' - 4y' + 4y = 0$$

$$y'' - 13y' + 42y = 0$$

$$y'' + 9y = 0$$