MAT 211: Linear Algebra

Practice Midterm 2

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Problem 1. Solve the following system of linear equations

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 4 & 10 \\ 1 & 3 & 6 & 10 & 20 \\ 1 & 4 & 10 & 20 & 35 \end{array}\right].$$

Problem 2. Give bases for $\mathrm{row}(A), \mathrm{col}(A), \mathrm{null}(A)$, where

1) $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$,

2) $A = \begin{bmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{bmatrix}$.

Problem 3. Find all possible values of $\mathrm{rank}(A)$ as a varies

$$A = \begin{bmatrix} 1 & 2 & a \\ -2 & 4a & 2 \\ a & -2 & 1 \end{bmatrix}.$$

Problem 4. Find all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Problem 5. **(a)** Find a basis for the minimal subspace in $\mathbb{R}^4$ containing the points $(1,-1,0,0)$, $(0,1,0,-1)$, $(0,0,-1,1)$, $(-1,0,1,0)$.

(b) Find a basis for the minimal subspace in $\mathbb{R}^3$ containing the point $(0,1,1)$ and the line

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Problem 6. Consider a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfying

$$T\left(\begin{bmatrix}-1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\-2\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}3\\2\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}.$$

Find the standard matrix of T

Problem 7. Let u, v be a basis for $\mathbb{R}^2$. Show that

1) $u + v, u + v$ is not a basis for $\mathbb{R}^2$;

2) $u + v, v$ is a basis for $\mathbb{R}^2$;

3) $u + v, u - v$ is a basis for $\mathbb{R}^2$.

Problem 8. Are the following transformations linear?

1) $T\begin{bmatrix}x\\y\end{bmatrix} = x\begin{bmatrix}1\\2\end{bmatrix} + 7\begin{bmatrix}x-y\\3\end{bmatrix}$,

2) $K\begin{bmatrix}x\\y\end{bmatrix} = x\begin{bmatrix}1\\2+y\end{bmatrix} + \begin{bmatrix}y\\x\end{bmatrix}$,

3) $S\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}y\\|x|\end{bmatrix}$.

Problem 9. Let F be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that F reflects a vector in the x-axis. Compute the standard matrix of F.

Problem 10. Compute the determinant of

$$A = \begin{bmatrix}1 & -1 & 0 & 3\\2 & 5 & 2 & 6\\0 & 1 & 0 & 0\\1 & 4 & 2 & 1\end{bmatrix}.$$

Problem 11. Is the matrix

$$A = \begin{bmatrix}2 & 3 & 0\\1 & -2 & -1\\2 & 0 & -1\end{bmatrix}$$

invertible? If yes, compute the inverse of A.

Problem 12. Find all a such that the matrix

$$A = \begin{bmatrix}a & 0 & 0\\1 & a & 0\\0 & 1 & a\end{bmatrix}$$

is invertible.