

MAT 211: Linear Algebra
Practice Midterm 1

Stony Brook University
Dzmitry Dudko

Fall 2021

Problem 1. Check if $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 6 \\ -3 \end{bmatrix}$ are orthogonal vectors.

Problem 2. In the following problems compute $u \cdot v$.

1) $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

2) $u = \begin{bmatrix} 1 \\ x \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 3 \\ y \end{bmatrix}$

3) $u = \begin{bmatrix} x \\ 2 \\ -3 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 4 \\ x \end{bmatrix}$

Problem 3. In the following problems find all k such that u and v are parallel vectors.

1) $u = \begin{bmatrix} k \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

2) $u = \begin{bmatrix} k \\ 1 \\ k \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3) $u = \begin{bmatrix} k \\ 0 \\ -k \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

Problem 4.

- 1) Find the general and parametric equations of the line passing through the points $(3, 1)$ and $(1, 0)$.
- 2) Are the points $(2, 1)$, $(1, 2)$, and $(4, -1)$ on the same line? If yes, find the general and parametric equations of the line passing through these points.
- 3) Find the parametric equation of the line passing through $(1, 1)$ and $(0, x)$.

Problem 5. Find the general and parametric equations of the plane passing through the point $(1, 1, 1)$ and orthogonal to the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Problem 6. Find the general and parametric equations of the plane passing through the points $(0, 1, 2)$, $(1, 0, 1)$, and $(2, 1, 4)$. Does the plane also pass through the origin $(0, 0, 0)$?

Problem 7. Solve the following system of linear equations:

$$\begin{aligned}x + 2y - 3z &= 9, \\2x - y + z &= 0, \\4x - y + z &= 4.\end{aligned}$$

Problem 8. Solve the following system of linear equations:

$$\left[\begin{array}{cccc|c} -1 & 3 & -2 & 4 & 0 \\ 2 & -6 & 1 & -2 & -3 \\ 1 & -3 & 4 & -8 & 2 \end{array} \right]$$

Problem 9. Determine if the following vectors are linearly independent

$$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}.$$

Problem 10. Find all k such that the following vectors are linearly independent

$$\begin{bmatrix} 2k \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Problem 11. Check if the span of the following vectors is \mathbb{R}^3 .

$$\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -4 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 7 \\ 6 \\ 0 \end{bmatrix}.$$

Problem 12. Calculate the product

$$\begin{bmatrix} 2k & 1 \\ 1 & k \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$