## MAT 132: Calculus 2 Practice Problems for the Final

Stony Brook University

Fall 2021

It is also recommended to review Practice Problems for Midterms 1 and 2.

**Problem 1.** Compute the following integrals:

$$\int \left(e^x + e^{-x}\right)^2 dx$$

$$\int_0^{\pi/2} 3\cos^2 x \sin x dx$$

$$\int_0^\pi \left(\cos^2 x + \cos^2(2x)\right) dx$$

$$\int_0^1 x^2 e^{x^3} dx$$

$$\int \frac{2-x}{x(x+1)} dx$$

$$\int \ln(x^2 + x) dx$$

**Problem 2.** Let R denote the region in the plane bounded by the 4 curves x = 0,  $x = \pi$ , y = 0, and  $y = \sin x + 1$ .

(a) Compute the area of R.

(b) Compute the volume when R is rotated around the x-axis.

**Problem 3.** A particle is moving along the x-axis; its speed at any time  $t \ge 0$  is given in terms of t by the formula  $t^2 e^t$ .

Compute the total distance traveled by the particle during the time interval  $0 \le t \le 2$ .

**Problem 4.** For each of the following improper integrals, determine whether it converges or not. If the integral converges, then determine its value.

$$\int_{-1}^{2} \frac{dx}{x^{3}}$$
$$\int_{0}^{\infty} \frac{x}{x^{2}+1} dx$$
$$\int_{0}^{\infty} \frac{x}{(x^{2}+1)^{2}} dx$$
$$\int_{0}^{\infty} \sin^{2} x dx$$

**Problem 5.** A spring has a natural length of 10 cm. It takes 1 J to stretch the spring from 10 cm to 15 cm. How much work would it take to stretch the spring from 5 cm to 20 cm?

Problem 6. Find the limits of the following sequences:

a) 
$$\lim_{n \to \infty} \frac{3 - n^2}{n^3 - n(n^2 - 1)},$$
 b) 
$$\lim_{n \to \infty} \frac{e^{1 - n}}{1 + n}$$
  
c) 
$$\lim_{n \to \infty} \frac{(1 + n!)^2}{(1 - n!)^2},$$
 d) 
$$\lim_{n \to \infty} \left( \frac{2^n}{1 + 2^{-n}} - 2^n \right)$$
  
e) 
$$\lim_{n \to \infty} \sqrt{\frac{n + 3^n}{3^n + 5}},$$
 e) 
$$\lim_{n \to \infty} \frac{5n!}{2^n + 1}.$$

**Problem 7.** Determine if the following series converge absolutely, converge conditionally, or diverge. No explanation is required in this problem.



Problem 8. Consider the following Maclaurin series

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$$

(a) Write the Maclaurin series for  $f(x) = \ln(1+2x)$  and for g(x) = f'(x).

(b) What is the radius of convergence for the series in (a)?

Problem 9. Consider the following Maclaurin series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

(a) Write the Maclaurin series for f(x) = x sin(x/5) and for g(x) = ∫ f(x)dx.
(b) What is the radius and interval of convergence for the series in (a)?

Problem 10. Find the general solutions to the following differential equations

$$\frac{dy}{dt} = 2\cos(2t+1)y$$
$$x^2y' = (x+1)y$$
$$y' = e^{x+y}$$
$$y' = x^2e^y$$

**Problem 10.** Solve the following initial-value problems with the initial condition y(0) = 1

$$y' = y + 1$$

$$y' = xy$$

**Problem 11.** Match the differential equations with corresponding direction vector fields. No explanation is required in this problem.

$$y' = x/y,$$
  $y' = y(3 - y),$   $y' = x^2 - y^2$   
 $y' = 2x - y,$   $y' = -2,$   $y' = 1$   
 $y' = \sin x \cos x,$   $y' = \sin y,$   $y' = |x|$ 

(One equation is without a direction vector field.)

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**Problem 12.** Find the general solutions to the following second order differential equations

$$y'' - 4y' + 4y = 0$$
$$y'' - 13y' + 42y = 0$$
$$y'' + 9y = 0$$