Problem 1. Let $f(x) = \dfrac{e^x + e^{-x}}{2}$.

(a) Find the average value of f on the interval $[-1, 1]$.

(b) Find the length of $y = f(x)$ from $x = -1$ to $x = 1$.

Problem 2. Solve the differential equation

$$y' = \frac{y \cos x}{1 + y^2}$$

subject to the initial condition $y(0) = 1$.

Problem 3. Sketch the solution curves to the differential equation $y' = -\frac{x}{y}$.

Problem 4. True or False?

(a) If y is any solution to $y' = x^4 + y^4 + y^2(1-x)$, then y is increasing when $x < 1$.

(b) If W is the solution to the differential equation $\dfrac{dW}{dt} = \dfrac{3}{1000}W$ satisfying the initial condition $W(0) = 100$, then $W\left(\dfrac{1000}{3}\ln(5)\right) = 500$.

(c) If W is the solution to the differential equation $\dfrac{dW}{dt} = \dfrac{3}{1000}W\left(1 - \dfrac{W}{400}\right)$ satisfying the initial condition $W(0) = 100$, then $\lim_{t\to\infty} W(t) = 400$.

(d) If C is onc solution curve for the differential equation $y' = \dfrac{x^3 + 2x - e^x}{1 + e^x}$, then infinitely many other solution curves can be obtained by shifting C to the right or left.

(e) If C is onc solution curve for the differential equation $y' = \dfrac{y^3 + 2y - e^y}{1 + e^y}$, then infinitely many other solution curves can be obtained by shifting C to the right or left.

Problem 5. Consider the system of differential equations governing the population of aphids and ladybugs:

$$\frac{dA}{dt} = 2A\left(1 - \frac{1}{10000}A\right) - \frac{1}{100}AL$$
$$\frac{dL}{dt} = -\frac{1}{2}L + \frac{1}{10000}AL$$

(a) Describe what will happen to the population of aphids in the absence of ladybugs.

(b) Find the equilibrium solution of the system.

(c) Find an expression for $\frac{dL}{dA}$.

Problem 6. Use Euler's method with step size 0.2 to approximate $y(0.4)$ where $y(x)$ is the solution to the differential equation $y' = 2xy^2$ with the initial value $y(0) = 1$.

Problem 7. Find a formula for the general term a_n of the sequence assuming that the general pattern of the first few terms continues:

(a) $\{1, 6, 11, 16, \ldots\}$

(b) $\left\{\frac{3}{4}, \frac{9}{16}, -\frac{27}{64}, \ldots\right\}$

Problem 8. Does the series $\sum_{n=1}^{\infty} \frac{n}{2n+1}$ converge or diverge? Explain.

Problem 9. Find the constant A so that

$$\sum_{k=2}^{\infty} A \left(\frac{2}{9}\right)^k = 5.$$

EXAM

Practice Midterm 2

Math 132

March 20, 2004