READING SEMINAR ON PERIOD IMAGES AND BAILY-BOREL COMPACTIFICATIONS

SPRING 2026

The goal of the seminar is to understand the following two papers:

(1) Bakker, Brunebarbe, Tsimerman, *O-minimal GAGA and a conjecture of Griffiths* (`https://arxiv.org/abs/1811.12230`)
(2) Bakker, Filipazzi, Mauri, Tsimerman, *Baily-Borel compactifications of period images and the b-semiampleness conjecture* (`https://arxiv.org/abs/2508.19215`)

The first paper proves that the image of any period map (associated to a polarized integral variation of Hodge structure) is a quasi-projective algebraic variety. The second paper constructs a functorial compactification for such period images and proves, along the way, that certain line bundles defined from Hodge theory are semiample. The main new tool in both is the definability of period maps and the theory of o-minimal structures.

The plan is to meet (almost) every week of the semester, ideally on Monday afternoon. This gives us a total of 14–15 talks. After looking in some detail at both papers, I propose to divide these roughly as follows.

(1) 6 talks about [BBT].
(2) 7–9 talks about [BFMT]

We can add more background material as needed, and speed up or slow down along the way.

Tentative list of talks

1. O-minimality and the definable Chow theorem. Give a brief review of o-minimal structures, especially the examples of $\mathbb{R}_{\text{alg}}$ and $\mathbb{R}_{\text{an,exp}}$. Summarize the main results, such as cell decomposition, dimension, closures. State the definable Chow theorem of Peterzil-Starchenko and sketch the proof. The reference is Peterzil and Starchenko, *Complex analytic geometry and analytic-geometric categories.*

2. Definable analytic spaces and coherent sheaves. Introduce definable analytic spaces and definable coherent sheaves (using the definable site). Sketch the proof of Oka coherence in this setting. This is most of Chapter 2 in [BBT], except for the sections on quotients and étale descent (which belong to the next talk).

3. Definabilization and definable GAGA. Review the definabilization of an algebraic space (as in 2.12), going back to the sections on quotients and étale descent as needed. Present the definable GAGA theorem (Theorem 3.1) and its proof. This is basically Chapter 3 of [BBT].

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4. Definable images. Present Theorem 4.2 and its proof; this is Chapter 4 in [BBT]. Discuss the example in 4.5, and try to explain how definability gets around such issues.

5. Definability of period maps. Briefly review period maps (for polarized integral variations of Hodge structure) and their definability. The relevant paper is Bakker, Klingler, Tsismerman, *Tame topology of arithmetic quotients and algebraicity of Hodge loci*, and maybe a little bit of background about degenerating variations of Hodge structure (such as the nilpotent orbit theorem) that is used to prove the definabiliy. (I've read that paper pretty carefully, so I can give advice to whoever choses this topic.) Deduce that images of period maps are algebraic varieties.

6. Period images are quasi-projective. Discuss the quasi-projectivity criterion in Chapter 5 of [BBT]. Introduce the Griffiths bundle, and then apply the criterion, as in Chapter 6, to show that period images are quasi-projective and that the Griffiths bundle is ample.

7. Overview of Baily-Borel compactifications. State the main result about Baily-Borel compactifications of period images, Theorems 1.1 and 1.2 in [BFMT], including semi-ampleness of the Griffiths bundle and finite generation of the ring of sections with moderate growth. Give a sketch of the proof, following the proof outline in 1.3.

8. Hodge theory background. Define the Griffiths bundle and Hodge bundles (Section 2.4). State and prove Theorem 2.22, which says that the Griffiths bundle has torsion combinatorial monodromy. State and prove the result (Lemma 2.19) about integrability of the Griffiths bundle. Review the variations of mixed Hodge structure on boundary strata (in a normal crossing compactification), as in 2.5. Discuss the minimal CY quotient, as in 2.6.

9. Quotient spaces. Present the material from Chapter 3 of [BFMT] about equivalence relations and quotient spaces, including the final algebraization result.

10. Semiampleness. Present the general semiampleness result for Hodge bundles (in Theorem 4.1) and the Griffiths bundle (Corollary 4.2). Explain the example in 4.2 that shows why the conditions in the theorem do not hold for arbitrary CY variations, but need geometric input.

11. The Baily-Borel compactification. Discuss the material about Baily-Borel compactifications of period images from Chapter 5 of [BFMT], especially the main result (Theorem 5.2), a give a sketch of the proof.

12. Birational geometry and Hodge theory of LC-trivial fibrations. This is Chapter 6 in [BFMT]. Define lc-trivial fibrations and state the canonical bundle formula (Theorem 6.12), especially the moduli part. Relate the moduli part to the Hodge bundle (in an Ambro model). Discuss sources and $\mathbb{P}^1$-linking (especially Kollár's Proposition 6.8). State and prove Theorem 6.31 about variations of Hodge structure associated to degenerations of CY pairs. This may need two talks.

13. b-semiampleness and applications. This is Chapter 7 in [BFMT]. State the b-semiampleness conjecture. Explain why, for lc-trivial fibrations, the two conditions (integrability and torsion combinatorial monodromy) are satisfied. Discuss at least the application to moduli of Calabi-Yau varieties in 7.3. Again, this may need two talks.