Math 589
Problem Set 6

due Wednesday, April 8, 2026

Most problems are from Hartshorne, *Algebraic Geometry*.

1. Use `Macaulay2` to solve this problem. The image of

$$\mathbb{P}^1 \to \mathbb{P}^6, \quad [t_0, t_1] \mapsto [t_0^6, t_0^5 t_1, \dots, t_1^6]$$

is a curve in $\mathbb{P}^6$. Any line between two distinct points on this curve is called a secant line; the secant variety X is (the closure of) the union of all the secant lines. Show that X is an irreducible projective variety of dimension 3, and determine generators for the homogeneous ideal $I(X)$. (Instead of writing down the generators, just say what the minimal number of generators is.)

2. Let $f\colon X \to Y$ be a morphism of schemes, and let $y \in Y$ be a point. Show that the underlying topological space of the fiber $X_y = \operatorname{Spec} k(y) \times_Y X$ is homeomorphic to $f^{-1}(y)$ with the subspace topology.

3. Show that if $i\colon Z \to X$ is a closed embedding, and $X' \to X$ is any morphism, then $i'\colon Z \times_X X' \to X'$ is also a closed embedding.

4. Problem II.2.16 in Hartshorne

5. Problem II.2.17 in Hartshorne

6. Problem II.3.11(b) in Hartshorne