

MATH 546, HOMEWORK 4

These problems are from Krantz, *Function Theory of Several Complex Variables*.

1. Let $D \subseteq \mathbb{R}^{n+1}$ be a bounded domain whose boundary $M = \partial D$ is a C^k -manifold for some, $k \geq 2$. Show that the function

$$\delta_M(x) = \begin{cases} -d_D(x) & \text{if } x \in \bar{D}, \\ d_D(x) & \text{if } x \notin D \end{cases}$$

is in $C^k(U)$ for a suitable open set U containing M . (This is Exercise 4 in Chapter 4 of Krantz' book; you can find a detailed outline there.)

2. Give a suitable definition for the “polynomial hull” of a set. If K is a compact subset of the unit disk in $\mathbb{C}$, then what is its polynomial hull?

3. Let D be a strongly (Levi) pseudoconvex domain in $\mathbb{C}^n$. Let $\phi: \Delta \rightarrow \bar{D}$ be a holomorphic mapping such that $\phi(0) \in \partial D$. Prove that ϕ must be constant.

4. Show that if $D_1 \subseteq D_2 \subseteq \cdots$ are pseudoconvex domains in $\mathbb{C}^n$, then their union is also pseudoconvex.

5. Consider the Hartogs domain

$$D = \{ (z_1, z_2) \in \mathbb{C}^2 \mid 1/2 < |z_1| < 1 \text{ or } 1/2 < |z_2| < 1 \}.$$

Let $K = \{ (0, z_2) \mid |z_2| = 3/4 \}$. Compute the hull of K with respect to the family of holomorphic function $\mathcal{O}(D)$, and conclude that it is not compact.