

MATH 546, HOMEWORK 3

These problems are from Krantz, *Function Theory of Several Complex Variables*.

1. Hartog's theorem. Read the proof of Hartog's theorem on separate analyticity in §2.4 of the book by Krantz. (The result is in Theorem 2.4.1.)

2. Let $D = \{ (z_1, z_2) \in \mathbb{C}^2 \mid |z_1| < |z_2| < 1 \}$ be the so-called Hartogs triangle. Prove that there is no bounded psh function u on D such that $u(z)$ tends to 0 as z tends to ∂D .

3. Prove that a domain $D \subseteq \mathbb{R}^n$ is convex if and only if the function $-\log d_D$ is a convex function on D ; here d_D is the Euclidean distance to ∂D .

4. Compute the Levi form for the surface defined by

$$\rho(z) = 2 \operatorname{Re} z_2 + |z_1|^8 + \lambda |z_1|^6 \operatorname{Re}(z_1^2).$$

What range of $\lambda \in \mathbb{R}$ will make the surface pseudoconvex near the origin?