HOMEWORK 6

Problems from the textbook

Atiyah-Macdonald, Ch. 2, #25, #26

Other problems (from Weibel's book)

1. Let $f\colon A_\bullet \to B_\bullet$ be a morphism of complexes; for simplicity, we denote all the differentials by the single letter d.

(a) The mapping cone $C_\bullet$ is the complex whose terms are $C_n = B_n \oplus A_{n-1}$ and whose differentials are

$$d\colon C_n \to C_{n-1}, \quad d(b,a) = \big(d(b) + f(a), -d(a)\big).$$

Check that this is indeed a complex.

(b) Show that we have a short exact sequence of complexes

$$0 \longrightarrow B_\bullet \xrightarrow{i} C_\bullet \xrightarrow{p} A_{\bullet-1} \longrightarrow 0,$$

if we use the differential $-d$ in the complex $A_{\bullet-1}$.

(c) Show that the connecting morphism δ in the long exact sequence is exactly the morphism $H_n(f)\colon H_n(A_\bullet) \to H_n(B_\bullet)$.

2. Show that a morphism of complexes $f\colon A_\bullet \to B_\bullet$ is homotopic to zero if and only if there is a morphism of complexes $s\colon C_\bullet \to B_\bullet$ from the mapping cone such that the composition $s \circ i\colon B_\bullet \to B_\bullet$ is the identity.

3. Suppose that A is a domain with fraction field F. Let M be an A-module. Show that $\mathrm{Tor}_1(F/A, M)$ is isomorphic to the torsion submodule

$$\big\{\, m \in M \;\big|\; \text{there is } a \neq 0 \text{ such that } am = 0 \,\big\}.$$

Due on Thursday, November 12.