HOMEWORK 5

PROBLEMS FROM THE TEXTBOOK

Atiyah-Macdonald, Ch. 9, #1, #2, #7

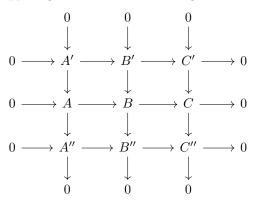
OTHER PROBLEMS (FROM WEIBEL'S BOOK)

1. Let **Groups** be the category of groups, and **Rings** the category of commutative rings with 1, with the obvious morphisms.

- (a) Show that a morphism in **Groups** is a monomorphism if and only if it is injective (as a map of sets).
- (b) Show that the inclusion $\mathbb{Z} \to \mathbb{Q}$ is an epimorphism in Rings.
- (c) Show that a monomorphism in **Groups** is a kernel if and only if its image (as a map of sets) is a normal subgroup.

2. Let $0 \to A \to B \to C \to 0$ be a short exact sequence of complexes. Show that if two of the three complexes A, B, C are exact, then so is the third.

3. $(3 \times 3 \text{ lemma})$ Suppose given a commutative diagram



in the category of R-modules, such that every column is exact. Show the following:

- (a) If the bottom two rows are exact, so is the top row.
- (b) If the top two rows are exact, so is the bottom row.
- (c) If the top and bottom rows are exact, and the composite $A \to C$ is zero, the middle row is also exact.
- (*Hint:* Show the remaining row is a complex, and apply the preceding exercise.)

Due on Thursday, October 29.