HOMEWORK 5

Problems from the textbook

Atiyah-Macdonald, Ch. 9, #1, #2, #7

Other problems (from Weibel's book)

1. Let `Groups` be the category of groups, and `Rings` the category of commutative rings with 1, with the obvious morphisms.

(a) Show that a morphism in `Groups` is a monomorphism if and only if it is injective (as a map of sets).
(b) Show that the inclusion $\mathbb{Z} \to \mathbb{Q}$ is an epimorphism in `Rings`.
(c) Show that a monomorphism in `Groups` is a kernel if and only if its image (as a map of sets) is a normal subgroup.

2. Let $0 \to A \to B \to C \to 0$ be a short exact sequence of complexes. Show that if two of the three complexes A, B, C are exact, then so is the third.

3. (3×3 lemma) Suppose given a commutative diagram

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A'' & \longrightarrow & B'' & \longrightarrow & C'' & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
$$

in the category of R-modules, such that every column is exact. Show the following:

(a) If the bottom two rows are exact, so is the top row.
(b) If the top two rows are exact, so is the bottom row.
(c) If the top and bottom rows are exact, and the composite $A \to C$ is zero, the middle row is also exact.

(*Hint:* Show the remaining row is a complex, and apply the preceding exercise.)

Due on Thursday, October 29.