# HOMEWORK 4

## Problems from the textbook

Atiyah-Macdonald, Ch. 5, #1, #4, #12, #13

## Other problems

**1.** Let $A$ be an artinian ring, $P \subseteq A$ a prime ideal. Prove that the homomorphism

$$A \to A_P, \quad a \mapsto a/1,$$

is surjective. (Hint: First show that if $s \notin P$, there is an element $a \notin P$ such that $as - 1 \in P$. Then argue that, for some $n \geq 1$, the element $(as-1)^n$ maps to 0 in the localization $A_P$. Deduce that $1/s$ is the image of an element in $A$.)

**2.** Let $A$ be an artinian ring, and let $P_1, \ldots, P_n$ be the finitely many distinct prime ideals of $A$. The purpose of this problem is to show that the homomorphism

$$\phi \colon A \to \prod_{i=1}^{n} A_{P_i}, \quad \phi(a) = (a/1, \ldots, a/1),$$

is an isomorphism, and hence that $A$ is a finite product of *local* artinian rings.

(a) Prove that $\phi$ is injective.
(b) Show that there are elements $a_1, \ldots, a_n \in A$ with the property that $a_i \in P_i$ and $a_i \notin P_j$ for $i \neq j$. (Hint: Use prime avoidance.)
(c) Show that there are elements $b_1, \ldots, b_n \in A$ with the property that $b_i \notin P_i$ and $b_i \in P_j$ for $i \neq j$.
(d) Prove that $\phi$ is surjective. (Hint: Use #1.)

Due on Tuesday, October 13.