HOMEWORK 3

Problems from the textbook

Atiyah-Macdonald, Ch. 7, #5; Ch. 8, #2, #3

Other problems

1. Let I be an ideal in a noetherian ring A, and let $P_1, \ldots, P_n$ be the finitely many associated primes of A/I. For every $j = 1, \ldots, n$, define an ideal

$$I_j = \left\{ a \in A \;\middle|\; \text{there exists } s \notin P_j \text{ such that } sa \in I \right\}$$

Prove that $I = I_1 \cap \cdots \cap I_n$.

2. Let k be a field. A monomial ideal is an ideal in $k[x_1, \ldots, x_n]$ that is generated by monomials in the variables $x_1, \ldots, x_n$. Which monomial ideals are (a) prime ideals, (b) primary ideals, (c) radical ideals?

3. Let M be a finitely generated module over the ring of integers $\mathbb{Z}$. Describe the set of associated primes of M in terms of the usual structure theory for finitely generated abelian groups.

4. Let A be a ring and $P \subseteq A$ a prime ideal. The n-th symbolic power of P is defined to be the ideal

$$P^{(n)} = \left\{ a \in A \;\middle|\; sa \in P^n \text{ for some } s \notin P \right\}.$$

Prove that $P^{(n)}$ is a P-primary ideal.

Due on Thursday, October 1.