Math 535
Problem Set 9

due Thursday, April 18, 2024

You may discuss problems with other students, but please write up your solutions on your own. Please try to write neatly. It is helpful if you staple all the pages together, and write your name on the first page.

1. Let M and N be two A-modules. Show that there is a unique isomorphism of A-modules $M \otimes N \cong N \otimes M$ that sends $m \otimes n$ to $n \otimes m$.

2. Let M, N, P be A-modules. Show that there is a unique function

$$\phi\colon \operatorname{Hom}_A\big(M \otimes_A N, P\big) \to \operatorname{Hom}_A\big(M, \operatorname{Hom}_A(N, P)\big)$$

that sends a morphism of A-modules $f\colon M \otimes_A N \to P$ to the morphism of A-modules $\phi(f)\colon M \to \operatorname{Hom}_A(N, P)$ with $\phi(f)(m)(n) = f(m \otimes n)$. Prove that ϕ is an isomorphism of A-modules.

3. Suppose we have an exact complex of A-modules

$$\cdots \to M \xrightarrow{f} N \xrightarrow{g} P \xrightarrow{h} Q \to \cdots$$

Show that g is injective if and only if $f = 0$, and that g is surjective iff $h = 0$. Conclude that g is an isomorphism iff $f = h = 0$.

Problems from the textbook

All problems are from the 3rd edition of *Abstract Algebra* by Dummit and Foote.

1. From Section 10.4, 4, 5, 6, 7, 11, 12