

**Math 535**  
**Problem Set 5**

due Thursday, February 29, 2024

You may discuss problems with other students, but please write up your solutions on your own. Please try to write neatly. It is helpful if you staple all the pages together, and write your name on the first page.

1. Let  $k \subseteq E$  be a finite Galois extension, and let  $f(x) \in k[x]$  be an irreducible polynomial of degree  $n$ . Show that  $f(x)$  factors in the polynomial ring  $E[x]$  into a product of  $m$  irreducible polynomials of degree  $d$ , with  $dm = n$ . Can you find an interpretation for  $m$  and  $d$ ?
2. Let  $\mathbb{Q} \subseteq E$  be the splitting field of a polynomial  $f(x) \in \mathbb{Q}[x]$  of degree 5. Suppose that  $\text{Gal}(E/\mathbb{Q})$  has order greater than 24. Show that  $f(x)$  must be irreducible.

**Problems from the textbook**

All problems are from the 3rd edition of *Abstract Algebra* by Dummit and Foote.

1. From Section 14.4, 1, 2, 3, 9
2. From Section 14.7, 3, 12