Math 535
Problem Set 4

due Thursday, February 22, 2024

You may discuss problems with other students, but please write up your solutions on your own. Please try to write neatly. It is helpful if you staple all the pages together, and write your name on the first page.

1. Let k be a field with infinitely many elements. Suppose that $k \subseteq E$ is a field extension with the property that there are only finitely many subfields $k \subseteq F \subseteq E$. Show that there is an element $\alpha \in E$ such that $E = k(\alpha)$. This result is called the "'primitive element theorem". (Hint: For $\alpha, \beta \in E$, consider the subfields $k(\alpha + c\beta)$ with $c \in k$.)

2. Let k be a field with infinitely many elements, and let $k \subseteq E$ be a finite Galois extension. Show that there is an element $\alpha \in E$ such that $E = k(\alpha)$.

Problems from the textbook

All problems are from the 3rd edition of *Abstract Algebra* by Dummit and Foote.

1. From Section 14.2, 3, 5, 7, 10, 15, 16