Math 535
Problem Set 3

due Thursday, February 15, 2024

You may discuss problems with other students, but please write up your solutions on your own. Please try to write neatly. It is helpful if you staple all the pages together, and write your name on the first page.

1. Let $k \subseteq E$ be a finite Galois extension of degree n, and set $G = \mathrm{Aut}_k(E)$. Show that every k-linear map $f\colon E \to E$ can be uniquely written in the form
$$f(e) = \sum_{i=1}^{n} a_i g_i(e),$$
where $a_1, \ldots, a_n \in k$ and $G = \{g_1, \ldots, g_n\}$.

2. Let $k \subseteq E$ be the splitting field of a polynomial $f(x) \in k[x]$. Show that if $g \in \mathrm{Aut}_k(E)$ is an automorphism that fixes all the roots of $f(x)$, then $g = \mathrm{id}$.

Problems from the textbook

All problems are from the 3rd edition of *Abstract Algebra* by Dummit and Foote.

1. From Section 13.6, 6, 8

2. From Section 14.2, 1, 2, 13, 14

3. From Section 14.3, 8

Note: If $k \subseteq E$ is a Galois extension, the group $\mathrm{Aut}_k(E)$ is called the *Galois group.*