Math 535
Additional Problem Set

due Tuesday, May 7, 2024

You may discuss problems with other students, but please write up your solutions on your own. Please try to write neatly. It is helpful if you staple all the pages together, and write your name on the first page.

1. Let k be a field of characteristic 3. Consider the subfield

$$F = \{\, f(t) \in k(t) \mid f(t) = f(2t+1) \,\}$$

   of the field of rational functions $k(t)$. Show that $k(t)$ is a Galois extension of F, and compute the Galois group.

2. Compute the Galois group of the polynomial $x^3 - 3x + 1 \in \mathbb{Q}[x]$. You may use without proof the fact that the discriminant of a cubic polynomial of the form $x^3 + px + q$ is equal to $-4p^3 - 27q^2$.

3. Let $\mathbb{Q} \subseteq K$ be a Galois extension such that $\mathrm{Gal}(K/\mathbb{Q})$ is cyclic of order 4. Prove that $i \notin K$.

4. Let $\alpha = \sqrt{5+\sqrt{5}}$.
   (a) Determine the minimal polynomial of α over $\mathbb{Q}$ and its splitting field E (as a subfield of $\mathbb{C}$).
   (b) Show that there is an automorphism g of the field E such that $g(\alpha) = \sqrt{5-\sqrt{5}}$ and $g(\sqrt{5}) = -\sqrt{5}$.
   (c) Prove that the Galois group $\mathrm{Gal}(E/\mathbb{Q})$ is cyclic.

Problems from the textbook

All problems are from the 3rd edition of *Abstract Algebra* by Dummit and Foote.

1. From Section 18.3, 2, 4, 5, 23