Math 535 Problem Set 1

due Thursday, February 1, 2024

You may discuss problems with other students, but please write up your solutions on your own. Please try to write neatly. It is helpful if you staple all the pages together, and write your name on the first page.

1. Another procedure for solving the quartic. Let $x^4 + ax^3 + bx^2 + cx + d = 0$ be a quartic equation, and denote by x_1, x_2, x_3, x_4 the four roots. Let $i = \sqrt{-1}$ and consider the two quantities

$$u = x_1 + ix_2 - x_3 - ix_4 = (x_1 - x_3) + i(x_2 - x_4)$$

$$v = x_1 - ix_2 - x_3 + ix_4 = (x_1 - x_3) - i(x_2 - x_4)$$

What happens to u and v under permutations of the roots? What about the quantity uv? Explain how one can use these observations to get formulas for the roots.

Problems from the textbook

All problems are from the 3rd edition of *Abstract Algebra* by Dummit and Foote.

- 1. From Section 13.1, 1–3
- 2. From Section 13.2, 3, 7, 14, 16
- 3. From Section 13.4, 3